PRINCIPLES *of* CORPORATE FINANCE

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RICHARD A. BREALEY

Professor of Finance
London Business School

STEWART C. MYERS

Robert C. Merton (1970) Professor of Finance
Sloan School of Management
Massachusetts Institute of Technology

FRANKLIN ALLEN

Nippon Life Professor of Finance
The Wharton School
University of Pennsylvania

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	Average Dividend Yield[a]	Forecasted Growth Rate[b]	Cost of Equity[c]
Burlington Northern Santa Fe	1.47%	12.74%	14.21%
CSX	1.06	15.52	16.58
Norfolk Southern	1.41	14.92	16.34
Union Pacific	1.91	12.67	14.57
Weighted average[d]			15.18

TABLE 5.3

Cost-of-equity estimates for U.S. railroads, 2005. The estimates use the constant-growth DCF model, which in this case probably overestimates the railroads' true cost of equity because the forecasted growth rates cannot be sustained in perpetuity.

[a] Average of monthly dividend yields during 2005.
[b] Based on IBES averages of security analysts' growth forecasts.
[c] Some rows do not add up because of rounding.
[d] Weights based on total market values of the railroads' common stock.
Source: U.S. Surface Transportation Board, "Railroad Cost of Capital—2005," September 15, 2006.

Dangers Lurk in Constant-Growth Formulas

The simple constant-growth DCF formula is an extremely useful rule of thumb, but no more than that. Naive trust in the formula has led many financial analysts to silly conclusions.

We have stressed the difficulty of estimating r by analysis of one stock only. Try to use a large sample of equivalent-risk securities. Even that may not work, but at least it gives the analyst a fighting chance, because the inevitable errors in estimating r for a single security tend to balance out across a broad sample.

In addition, resist the temptation to apply the formula to firms having high current rates of growth. Such growth can rarely be sustained indefinitely, but the constant-growth DCF formula assumes it can. This erroneous assumption leads to an overestimate of r. Table 5.3 is probably an example of such an overestimate. The four largest U.S. railroads were expanding rapidly in 2005 and 2006 as they recovered from a period of low profitability. Security analysts were forecasting continued recovery and earnings growth at 12% to 15% for the next few years. But the rate of growth was bound to slow down when the recovery was completed. Thus analysts and investors were not assuming a single future growth rate, but at least two: a near-term rate of rapid growth, then a transition to a moderate long-term growth rate. There was no basis for assuming 12% to 15% growth in perpetuity.

DCF Valuation with Varying Growth Rates Consider Growth-Tech, Inc., a firm with $DIV_1 = \$.50$ and $P_0 = \$50$. The firm has plowed back 80% of earnings and has had a return on equity (ROE) of 25%. This means that *in the past*

$$\text{Dividend growth rate} = \text{plowback ratio} \times \text{ROE} = .80 \times .25 = .20$$

The temptation is to assume that the future long-term growth rate g also equals .20. This would imply

$$r = \frac{.50}{50.00} + .20 = .21$$

Choice among methods of estimating share yield

The search for the growth component in the discounted cash flow model.

David A. Gordon, Myron J. Gordon, and Lawrence I. Gould

50 SPRING 1989

The yield at which a share of stock is selling, also called its expected return or required return, is an important statistic in finance. Firms use it in choosing among investment opportunities and financing alternatives, and investors use it in making portfolio decisions. Nevertheless, the yield at which a share is selling is a difficult quantity to measure, which has limited its use in the practice of finance. This paper develops and tests a basis for choice among alternative methods of estimating a share's yield.

A share's yield, like a bond's yield, is the discount rate that equates its expected future payments with its current price. A bond's yield is easy to measure under the common practice of ignoring default risk, as the future payments are then known with certainty. The future payments on a share, however, are dividends and market price, and these payments are uncertain.

The common practice is to represent these future dividend payments with estimates of two numbers: One is the coming dividend, and the other is a growth rate. The latter can be an estimate of the long-run growth rate in the dividend or of the growth rate in price over the coming period. In the latter case, the estimate is called the expected holding-period return (EHPR); in the former case, it is called the discounted cash flow yield (DCFY).[1] In either case, the estimate of a share's yield reduces to the sum of its dividend yield and a future growth rate, with the latter inferred in some way from historical data.

There is a wide variety of acceptable methods for using historical data to estimate future growth. This variation in method is illustrated in the testimony of expert witnesses before public utility commissions on the fair return for a public utility. In these cases, the estimates and the methods used are a matter of public record. Some idea of the various methods can be found in Morin (1984) and Kolbe, Read, and Hall (1984). The performance of alternative estimating methods has been examined in Gordon (1974), Kolbe, Read, and Hall (1984), Brigham, Shome, and Vinson (1985), and Harris (1986).

We have derived our basis for comparing the accuracy of alternative methods for estimating the DCFY on a share from the generally accepted propositions that yield should vary according to risk, and that beta is the best estimate of risk. Hence, the DCFY should vary among shares with beta, and, between two methods for estimating growth, the superior method is the one for which the variation in yield among shares is explained better by the variation in beta among the shares.

First we present simple, plausible, and objective measurement rules for implementing four popular and/or attractive methods for estimating the DCFY. We then describe how sample statistics may be used to judge the accuracy of each method. We also describe how the CAPM model has been used to estimate share yield and explain why we do not compare it with the various DCFY methods. The following section carries out the comparison with samples of utility and industrial shares, and the last section pre-

DAVID A. GORDON is in charge of transaction finance at Scotia McLeod, a subsidiary of the Bank of Nova Scotia in Toronto. MYRON J. GORDON is Professor of Finance at the Faculty of Management at the University of Toronto (Ontario M5S 1V4). LAWRENCE I. GOULD is Professor and Head of Accounting and Finance at the University of Manitoba in Winnipeg (Manitoba R3T 2N2).

sents the conclusions that may be drawn from the findings.

ALTERNATIVE MEASUREMENT RULES FOR A SHARE'S YIELD

Under the DCF method or model for estimating the expected return on a stock, the yield for the jth stock is:

$$DCFY_{jt} = DYD_{jt} + GR_{jt}, \quad (1)$$

where:

$DCFY_{jt}$ = DCF yield on the jth stock at time t,

DYD_{jt} = dividend yield on the jth stock at time t, and

GR_{jt} = long-run growth rate in the dividend on the jth stock that investors expect at time t.

In what follows, we omit the time and firm subscripts on the variables when they are not required. Also, DCFY will refer to the unknown true yield on a share.

The difficult problem in arriving at the DCFY is estimation of the long-run growth rate that investors expect. Four estimates of that quantity are:

EGR = rate of growth in earnings per share over a prior time period, usually the last five years;

DGR = rate of growth in dividend per share over a prior time period, usually the last five years;

FRG = consensus among security analyst forecasts of the growth rate in earnings, over the next five years; and

BRG = an average over the prior five years of the product of the retention rate b and rate of return on common equity r on a stock.

The estimate of share yield that incorporates each of these estimates of growth is denoted KEGR, KDGR, KFRG, and KBRG, respectively.

A case can be made for each of the four methods for estimating growth. KEGR, KDGR, and KBRG have been widely used in public utility testimony and in research on stock valuation models. The rationale for KEGR is the belief that the past growth rate in earnings is the best predictor of future growth in earnings and dividends. The rationale for KDGR is that the future growth rate in dividends is the statistic we want to estimate, and the past dividend record is free of the noise in past earnings.[2] The rationale for KBRG is that all variables will grow at this rate if the firm earns r and retains b. Furthermore, as Gordon and Gould (1980) show, KEGR and KDGR will be biased in one direction or another if r and b have changed over the last five years. As for KFRG, security analysts are professionals employed to forecast future performance; their forecasts are widely accepted by investors. The IBES collection of forecast growth rates of security analysts compiled by Lynch, Jones, and Ryan has increased the popularity of this estimate.

As stated earlier, we may also take the yield on a share as the sum of the dividend yield and the expected rate of growth in price over the coming period. This estimate of a share's yield is widely used in testing the CAPM, with the average HPR over the prior five years commonly used in such empirical work. On the other hand, this estimate of a share's yield varies so widely among firms and over time as to be patently in error as an estimate of share yield.[3]

BASIS OF COMPARISON

To compare the accuracy of the four estimates of the DCFY stated above, we regress the data under each estimate on beta for a sample of shares. If KEGR is the estimate,

$$KEGR_j = \alpha_0 + \alpha_1 BETA_j + \epsilon_j. \quad (2)$$

The rationale for this expression lies in the risk premium theory of share yield, where the share yield is equal to the interest rate plus a risk premium that varies with the share's relative risk. Hence, if BETA is an error-free index of relative risk, α_0 is equal to the interest rate, and α_1 is the risk premium on the market portfolio or standard share.[4]

The higher the correlation between KEGR and BETA, assuming that α_1 is positive, the greater the confidence we may have in KEGR as an estimate of DCFY. We cannot rely solely on the correlation, though, in selecting among the methods for estimating DCFY. Errors in KEGR as a basis for estimating the DCFY on the jth share have random and systematic components. The former is ϵ_j, and its average value can be taken as the root mean square error of the regression (MSE). The larger the root MSE of the regression, the less attractive KEGR is as an estimate of share yield, because the error makes the problem of choice between $KEGR_j$ and $KEGR_j - \epsilon_j$ more acute. (That problem will be discussed shortly.)

The systematic error is the difference between the unknown true yield on the jth share, $DCFY_j$, and the value predicted by Equation (2). There is no obvious measure of the systematic error, as we do not know $DCFY_j$, but sample values of α_0 may provide information on its average value. The difference between α_0 and the interest rate is an indicator of systematic error, because the difference is zero under the risk premium theory. Error in the measurement of BETA biases α_0 upward, but, with the same BETA for each share used in all four regressions, differences in α_0 are indicators of systematic error.[5]

In addition to regression statistics, the sample mean and standard deviation of KEGR is a source of information on its accuracy as a method for the estimation of DCFY. If the mean departs radically from the long-term bond rate, or if the standard deviation indicates an unreasonable range of variation among shares, the accuracy of the method is open to question. Also, the sample mean may be a source of information on the systematic error for a method of estimation. Hence, sample values for the mean, standard deviation, correlation, root MSE, and constant term all contribute to a judgment on a method's accuracy for estimating the DCFY on a share. Unfortunately, there is no simple criterion for choice among the alternatives.

Once a conclusion is reached on the most accurate method for estimating DCFY — say, KEGR — we then have the problem of choice between $KEGR_j$ and $KEGR_j - \epsilon_j$ for the jth share. If the random error in $KEGR_j$ is due to error in its measurement for the jth share, we simply use the value predicted by Equation (2), which is $KEGR_j - \epsilon_j$. On the other hand, KEGR and DCFY may vary among shares with other (omitted) variables as well as BETA, in which case ϵ_j is also due to the omitted variables, and $KEGR_j$ may be the better estimate of DCFY. Unfortunately, we have no basis for choice among these two hypotheses, and the smaller the root MSE the less troublesome the problem of choice between them.

A more favorable tax treatment of capital gains over dividends should make investors prefer capital gains to dividends. As Brennan (1973) has shown, the yield investors require on a share would then vary with the excess of its dividend yield over the interest rate. To recognize this, Equation (2) becomes

$$KEGR_j = \alpha_0 + \alpha_1 BETA_j + \alpha_2 DMI_j + \epsilon_j, \qquad (3)$$

with DMI_j the excess of the dividend yield over the interest rate for the jth firm. Although the tax effect should make α_2 positive, its information in DMI on share risk would tend to make α_2 negative. That is, dividend yield varies inversely with expected growth, and we would find α_2 negative insofar as growth is risky. To the extent that these two influences of the dividend yield offset each other, α_2 will tend toward zero.

The CAPM theory of how expected return varies among shares has been proposed as an alternative to the DCF model for measuring yield. Its value for the jth stock is

$$EHPR_j = INTR + BETA_j[EHPR_m - INTR], \qquad (4)$$

where:

$EHPR_j$ = expected holding-period return on the jth share,

$INTR$ = one-period risk-free interest rate

$EHPR_m$ = expected holding-period return on the market portfolio.

There is an important difference between this CAPM model of share yield and the DCF model represented by Equation (1). The latter is merely an instrument for measuring share yield: There is nothing in the DCF model that explains the variation in yield among shares. The CAPM, on the other hand, is a theory on why and how yield varies among shares, but one must go outside of the theory to estimate the variables on the right-hand side of Equation (4). Given rules for estimating the variables, EHPR and BETA, empirical work then provides a joint test of the theory and the estimating rules, such as we are carrying out here.[6]

The CAPM nonetheless has been used to estimate share yield in testimony before regulatory commissions by assigning numbers to each of the quantities on the right-hand side of Equation (4). For INTR, a long-term bond yield is sometimes used instead of a one-period rate. BETA is estimated by conventional methods.

The big problem is the expected return on the market portfolio. Here the practice has been to use the average realized risk premium over a period of about fifty years as the estimate of $EHPR_m - INTR$ in Equation (4). Although the implicit assumption is that the risk premium is a constant over time, we would expect the premium to change from one period to the next for various reasons, among them changes in the interest rate, the risk premium on the market portfolio, and the relative taxation of interest and share income. Hence, this estimate of share yield is more or less in error at any particular time, but we have no way of estimating this error and comparing the method with the others.

COMPARATIVE PERFORMANCE

We carried out our empirical work with a sample of 75 large electric and gas utility firms and a sample of 244 firms that includes 169 industrial firms drawn from the S&P 400. We obtained share yield under the four methods for estimating it as of the start of the year for the years 1984, 1985, and 1986.

For the explanatory variables, BETA for each share on each date was obtained by regressing the monthly HPRs for the share on the monthly HPRs for the S&P 500 over the prior five years. DMI for a share is its dividend yield less the interest rate on the one-month Treasury bill at the start of each year. EGR and DGR are the growth rates in earnings and in dividends per share, respectively, over the prior five years as reported on the Value Line Tape. BRG is a weighted average of the retention growth rates over the prior five years,[7] and FRG is the average of forecast growth rates in earnings over the next five years reported by IBES. The corresponding estimates of share yield were obtained by adding the dividend yield at the start of each year to the estimate of growth.

Table 1 presents the statistics that we obtained with KBRG and KFRG as the estimates of DCFY for the sample of utility shares and of all shares. The means of KBRG for the utility shares seems reasonable, with the interest rate on ten-year government bonds the standard of comparison, the latter being 11.67%, 10.43%, and 9.19% at the start of 1984, 1985, and 1986, respectively.[8] The standard deviations for KBRG are small enough to make its range of variation well within the bounds of reason. The lower means for all shares reveal that the means for industrial shares are below the means for utility shares.[9] This casts doubt on the accuracy of KBRG as a basis for estimating the DCFY on industrial shares, because industrials are riskier than utility shares.

The beta model explains none of the variation in KBRG among utility shares, but the two-factor model is a substantial improvement. The DMI coefficient, α_2, is positive and significant in every year, meaning that the unfavorable tax effect of a high dividend yield dominates the favorable risk effect. The coefficient on BETA is positive and significant in two of the three years. The only disturbing feature of the data is the sharp fall in R^2 and the corresponding rise in the root MSE relative to the standard deviation of KBRG as we go from 1984 to 1986.

The KBRG statistics for all shares are substantially inferior to the utility share statistics. This forces the unhappy conclusion that, for industrial shares, BETA is a poor measure of risk, or KBRG is a poor measure of DCFY, or both.

The KFRG statistics for the utility sample are superior to the KBRG statistics. The means are reasonable under the two criteria of being above the interest rate and moving with it. The range of variation of KFRG suggested by its standard deviations seems reasonable. The statistics for the beta model are a slight improvement on the corresponding statistics for KBRG. Furthermore, the two-factor model does a good job of explaining the variation in KFRG among

TABLE 1

Sample and Regression Statistics for KBRG and KFRG, Utility Shares and All Shares, 1984, 1985, and 1986

	KBRG			KFRG		
	1984	1985	1986	1984	1985	1986
			UTILITY SHARES (75)			
Mean	14.84	14.38	12.93	15.64	14.56	12.93
Standard Deviation	2.51	1.87	1.80	2.26	1.43	1.42
Beta Model α_0	14.26	13.96	13.05	15.14	13.48	12.74
α_1	1.44	1.21	−0.28	1.25	3.09	0.42
t-statistic	(0.97)	(1.12)	(0.19)	(0.93)	(4.14)	(0.37)
Root MSE	2.52	1.87	1.81	2.26	1.29	1.43
R^2	0.013	0.017	0.001	0.012	0.190	0.002
Two-Factor Model α_0	12.45	12.75	12.42	13.30	12.46	11.97
α_1	3.45	2.11	0.11	3.28	3.85	0.89
t-statistic	(3.13)	(2.19)	(0.08)	(3.83)	(6.33)	(0.88)
α_2	0.68	0.45	0.34	0.68	0.38	0.41
t-statistic	(8.22)	(4.88)	(2.81)	(10.73)	(6.52)	(4.65)
Root MSE	1.82	1.63	1.73	1.41	1.03	1.26
R^2	0.491	0.262	0.100	0.620	0.491	0.232
			ALL SHARES (244)			
Mean	12.98	13.19	11.86	16.17	15.87	14.31
Standard Deviation	3.86	3.21	3.52	2.60	2.32	2.30
Beta Model α_0	15.00	14.71	13.90	15.56	14.50	12.57
α_1	−2.47	−1.91	−2.40	0.74	1.72	2.05
t-statistic	(4.23)	(4.15)	(4.25)	(1.83)	(5.29)	(5.70)
Root MSE	3.73	3.10	3.40	2.59	2.20	2.16
R^2	0.069	0.066	0.069	0.014	0.104	0.118
Two-Factor Model α_0	14.34	14.42	13.95	15.40	14.61	12.75
α_1	0.09	−1.18	−2.51	1.37	1.44	1.61
t-statistic	(0.13)	(2.04)	(3.45)	(2.69)	(3.52)	(3.49)
α_2	0.48	0.17	−0.02	0.12	−0.06	−0.10
t-statistic	(6.04)	(2.09)	(0.24)	(2.01)	(1.12)	(1.53)
Root MSE	3.49	3.08	3.41	2.57	2.20	2.16
R^2	0.191	0.083	0.070	0.030	0.108	0.127

utility shares. The R^2s are higher here than for KBRG in every year. Finally, α_2 is positive and significant in every year, and α_1 is not significant only in 1986.

The implicit means of KFRG for the industrial shares seem high but not beyond reason. On the other hand, the regression statistics for the all-shares sample are not good, which leads to the same unhappy conclusion for industrial shares as we reached for KBRG.

Table 2 presents the statistics that we obtained using KEGR and KDGR as estimates of the DCFY on the shares in our samples. Comparison of the regression statistics with those in Table 1 reveals that KEGR and KDGR, particularly the former, fall short by a wide margin of the performance of KBRG and KFRG as estimates of the DCFY on a share.

CONCLUSION

We have compared the accuracy of four methods for estimating the growth component of the discounted cash flow yield on a share: past growth rate in earnings (KEGR), past growth rate in dividends (KDGR), past retention growth rate (KBRG), and forecasts of growth by security analysts (KFRG). Criteria for the comparison were the reasonableness of sample means and standard deviations and the success of beta and dividend yield in explaining the variation in DCF yield among shares. For our sample of utility shares, KFRG performed well, with KBRG, KDGR, and KEGR following in that order, and with KEGR a distant fourth. If we had used past growth in price, it would have been an even more distant fifth. Nevertheless, none of the four estimates of growth performed well under the criteria for a sample that included industrial shares.

Before closing, we have three observations to make. First, the superior performance by KFRG should come as no surprise. All four estimates of growth rely upon past data, but in the case of KFRG a larger body of past data is used, filtered through a group of security analysts who adjust for abnormalities that are not considered relevant for future growth. We assume this is done by any analyst who develops retention growth estimates of yield for a firm. If we had done this for all seventy-five firms in our utility sample, it is likely that the correlations

TABLE 2

Sample and Regression Statistics for KEGR and KDGR, Utility Shares and All Shares, 1984, 1985, and 1986

	KEGR			KDGR		
	1984	1985	1986	1984	1985	1986
			UTILITY SHARES (75)			
Mean	16.16	0.32	14.91	16.49	15.76	14.13
Standard Deviation	3.31	3.47	4.66	3.12	2.41	2.21
Beta Model α_0	15.45	16.18	0.51	15.75	14.53	12.30
α_1	1.75	0.40	−7.87	1.83	3.53	3.99
t-statistic	(0.89)	(0.20)	(2.16)	(0.99)	(2.64)	(2.32)
Root MSE	3.32	3.49	4.55	3.12	2.32	2.15
R^2	0.010	0.001	0.060	0.013	0.087	0.069
Two-Factor Model α_0	14.20	15.83	18.76	14.10	13.56	12.64
α_1	3.13	0.66	−8.03	3.65	4.25	3.78
t-statistic	(1.66)	(0.32)	(2.18)	(2.23)	(3.26)	(2.20)
α_2	0.47	0.13	−0.13	0.61	0.35	−0.18
t-statistic	(3.32)	(0.66)	(0.42)	(5.02)	(2.86)	(1.21)
Root MSE	3.11	3.50	4.58	2.70	2.21	2.14
R^2	0.142	0.007	0.063	0.269	0.180	0.087
			ALL SHARES (244)			
Mean	11.14	9.42	7.88	15.08	13.63	11.35
Standard Deviation	10.67	11.67	11.45	6.08	6.30	6.71
Beta Model α_0	15.96	18.28	19.55	15.15	0.04	15.39
α_1	−5.90	−11.16	−13.70	−0.09	−1.78	−4.74
t-statistic	(3.62)	(7.07)	(8.10)	(0.09)	(1.92)	(4.41)
Root MSE	10.41	10.65	10.18	6.09	6.27	6.47
R^2	0.051	0.171	0.213	0.000	0.015	0.074
Two-Factor Model α_0	14.84	18.01	19.91	14.31	14.11	14.79
α_1	−1.56	−10.49	−14.62	3.17	0.63	−3.25
t-statistic	(0.77)	(5.27)	(6.72)	(2.73)	(0.55)	(2.36)
α_2	0.81	0.15	−0.21	0.61	0.55	0.34
t-statistic	(3.51)	(0.55)	(0.67)	(4.57)	(3.47)	(1.72)
Root MSE	10.18	10.67	10.19	5.86	6.13	6.45
R^2	0.097	0.172	0.215	0.080	0.062	0.085

would have been as good or better than those obtained with the analyst forecasts of growth.

Second, we examined shares and not portfolios, because our objective is to estimate the DCFY for shares and not for portfolios. As common practice in testing the CAPM has been to execute tests on portfolios instead of shares, we classified our population of shares into ten portfolios on the basis of their beta values. Regression statistics were substantially unchanged, except that correlations increased dramatically.

Finally, we must acknowledge that we have no basis for estimating the expected HPR or DCF yield for industrial shares with any confidence. Theories on financial decision-making in industrial corporations that rely on that statistic have a weak empirical foundation.

[1] The EHPR is a one-period return, while the DCFY is a yield to maturity measure. The two may differ in actuality because of measurement problems, but they also may differ in theory. That is, they may differ in the same way that interest rates on bonds of different maturities may differ. See Gordon and Gould (1984a). This source of difference between EHPR and DCFY will be ignored here.

[2] A widely accepted hypothesis is that dividends contain information on earnings, because management sets the dividend to pay out a stable fraction of normal or permanent earnings.

[3] Over a five-year period, there may even be a negative rate of growth in price for a large number of firms. Furthermore, this negative growth rate may be larger in absolute value than the dividend yield, which leads to the conclusion that investors are holding such shares to earn a negative return. The frequency of negative rates of growth in price is reduced as the prior time period used in its calculation increases in length. As that takes place, however, the estimate of the expected return for a firm approaches a constant or a constant plus the dividend yield. The expected return on a share is one statistic for which it is an error to assume that expectations are on average realized.

[4] Equation (2) is similar to the CAPM according to Sharpe, Lintner, and Mossin. They arrived at this expression under very rigorous assumptions. The heuristic risk premium model is adequate for our purposes.

[5] It may be thought that Theil's (1966) decomposition of the difference between the actual and predicted values of a variable can be used here, but in fact that decomposition applies to a different problem. It assumes that the observed (actual) past values of a variable are free of error, and it decomposes the error in a model that is employed to explain the past values. The purpose of Theil's decomposition is to cast light on the possible error in using the model to predict future values of the dependent variable. Our problem is to determine which set of observed values is closest to the true values, with the risk premium theory of share yield and BETA as the source of information on the true values. Theil's method would be appropriate for decomposing the difference between the actual and predicted values of the *realized* holding-period return on a share. The actual values here can be observed without error.

[6] There is an enormous volume of empirical work devoted to discovering whether the theory is true, but this empirical work does not provide useful estimates of the EHPR on a share. To test the truth of Equation (4), the practice has been to regress EHPR on BETA for a sample of firms with the average realized HPR over the prior five or so years used as an estimate of the EHPR. Because of the large error in the realized HPR over a prior time period, as noted earlier, neither the actual values of the dependent variable nor the values predicted by the model are usable as estimates of share yield. See Fama and MacBeth (1973) and Friend, Westerfield, and Granito (1978).

[7] BRG for a year is earnings less dividend divided by the end-of-year book value. The estimate of the expected value as of the start of 1986 is 0.3BRG85 + 0.25BRG84 + 0.20BRG83 + 0.15BRG83 + 0.10BRG82. If any value of BRG was negative, it was set equal to zero.

[8] We expect the yields on shares to be above the risk-free interest rate, but with a high enough interest rate the more favorable tax treatment of shares can reduce the yield below the interest rate. Interest rates were not that high in these years. See Gordon and Gould (1984b).

[9] The statistics reported for all shares and for utility shares were also obtained for industrial shares. All methods of estimation performed so poorly for industrial shares, however, as to suggest no confidence can be placed in any of them. To save space, we do not present statistics for the industrial shares. Whatever we want to know about them can be deduced by comparing the data for all shares and utility shares.

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The Cost of Capital, Corporation Finance and the Theory of Investment

Franco Modigliani; Merton H. Miller

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THE COST OF CAPITAL, CORPORATION FINANCE AND THE THEORY OF INVESTMENT

By FRANCO MODIGLIANI AND MERTON H. MILLER*

What is the "cost of capital" to a firm in a world in which funds are used to acquire assets whose yields are uncertain; and in which capital can be obtained by many different media, ranging from pure debt instruments, representing money-fixed claims, to pure equity issues, giving holders only the right to a pro-rata share in the uncertain venture? This question has vexed at least three classes of economists: (1) the corporation finance specialist concerned with the techniques of financing firms so as to ensure their survival and growth; (2) the managerial economist concerned with capital budgeting; and (3) the economic theorist concerned with explaining investment behavior at both the micro and macro levels.[1]

In much of his formal analysis, the economic theorist at least has tended to side-step the essence of this cost-of-capital problem by proceeding as though physical assets—like bonds—could be regarded as yielding known, sure streams. Given this assumption, the theorist has concluded that the cost of capital to the owners of a firm is simply the rate of interest on bonds; and has derived the familiar proposition that the firm, acting rationally, will tend to push investment to the point

* The authors are, respectively, professor and associate professor of economics in the Graduate School of Industrial Administration, Carnegie Institute of Technology. This article is a revised version of a paper delivered at the annual meeting of the Econometric Society, December 1956. The authors express thanks for the comments and suggestions made at that time by the discussants of the paper, Evsey Domar, Robert Eisner and John Lintner, and subsequently by James Duesenberry. They are also greatly indebted to many of their present and former colleagues and students at Carnegie Tech who served so often and with such remarkable patience as a critical forum for the ideas here presented.

[1] The literature bearing on the cost-of-capital problem is far too extensive for listing here. Numerous references to it will be found throughout the paper though we make no claim to completeness. One phase of the problem which we do not consider explicitly, but which has a considerable literature of its own is the relation between the cost of capital and public utility rates. For a recent summary of the "cost-of-capital theory" of rate regulation and a brief discussion of some of its implications, the reader may refer to H. M. Somers [20].

where the marginal yield on physical assets is equal to the market rate of interest.[2] This proposition can be shown to follow from either of two criteria of rational decision-making which are equivalent under certainty, namely (1) the maximization of profits and (2) the maximization of market value.

According to the first criterion, a physical asset is worth acquiring if it will increase the net profit of the owners of the firm. But net profit will increase only if the expected rate of return, or yield, of the asset exceeds the rate of interest. According to the second criterion, an asset is worth acquiring if it increases the value of the owners' equity, *i.e.*, if it adds more to the market value of the firm than the costs of acquisition. But what the asset adds is given by capitalizing the stream it generates at the market rate of interest, and this capitalized value will exceed its cost if and only if the yield of the asset exceeds the rate of interest. Note that, under either formulation, the cost of capital is equal to the rate of interest on bonds, regardless of whether the funds are acquired through debt instruments or through new issues of common stock. Indeed, in a world of sure returns, the distinction between debt and equity funds reduces largely to one of terminology.

It must be acknowledged that some attempt is usually made in this type of analysis to allow for the existence of uncertainty. This attempt typically takes the form of superimposing on the results of the certainty analysis the notion of a "risk discount" to be subtracted from the expected yield (or a "risk premium" to be added to the market rate of interest). Investment decisions are then supposed to be based on a comparison of this "risk adjusted" or "certainty equivalent" yield with the market rate of interest.[3] No satisfactory explanation has yet been provided, however, as to what determines the size of the risk discount and how it varies in response to changes in other variables.

Considered as a convenient approximation, the model of the firm constructed via this certainty—or certainty-equivalent—approach has admittedly been useful in dealing with some of the grosser aspects of the processes of capital accumulation and economic fluctuations. Such a model underlies, for example, the familiar Keynesian aggregate investment function in which aggregate investment is written as a function of the rate of interest—the same riskless rate of interest which appears later in the system in the liquidity-preference equation. Yet few would maintain that this approximation is adequate. At the macroeconomic level there are ample grounds for doubting that the rate of interest has

[2] Or, more accurately, to the marginal cost of borrowed funds since it is customary, at least in advanced analysis, to draw the supply curve of borrowed funds to the firm as a rising one. For an advanced treatment of the certainty case, see F. and V. Lutz [13].

[3] The classic examples of the certainty-equivalent approach are found in J. R. Hicks [8] and O. Lange [11].

as large and as direct an influence on the rate of investment as this analysis would lead us to believe. At the microeconomic level the certainty model has little descriptive value and provides no real guidance to the finance specialist or managerial economist whose main problems cannot be treated in a framework which deals so cavalierly with uncertainty and ignores all forms of financing other than debt issues.[4]

Only recently have economists begun to face up seriously to the problem of the cost of capital *cum* risk. In the process they have found their interests and endeavors merging with those of the finance specialist and the managerial economist who have lived with the problem longer and more intimately. In this joint search to establish the principles which govern rational investment and financial policy in a world of uncertainty two main lines of attack can be discerned. These lines represent, in effect, attempts to extrapolate to the world of uncertainty each of the two criteria—profit maximization and market value maximization—which were seen to have equivalent implications in the special case of certainty. With the recognition of uncertainty this equivalence vanishes. In fact, the profit maximization criterion is no longer even well defined. Under uncertainty there corresponds to each decision of the firm not a unique profit outcome, but a plurality of mutually exclusive outcomes which can at best be described by a subjective probability distribution. The profit outcome, in short, has become a random variable and as such its maximization no longer has an operational meaning. Nor can this difficulty generally be disposed of by using the mathematical expectation of profits as the variable to be maximized. For decisions which affect the expected value will also tend to affect the dispersion and other characteristics of the distribution of outcomes. In particular, the use of debt rather than equity funds to finance a given venture may well increase the expected return to the owners, but only at the cost of increased dispersion of the outcomes.

Under these conditions the profit outcomes of alternative investment and financing decisions can be compared and ranked only in terms of a *subjective* "utility function" of the owners which weighs the expected yield against other characteristics of the distribution. Accordingly, the extrapolation of the profit maximization criterion of the certainty model has tended to evolve into utility maximization, sometimes explicitly, more frequently in a qualitative and heuristic form.[5]

The utility approach undoubtedly represents an advance over the certainty or certainty-equivalent approach. It does at least permit us

[4] Those who have taken a "case-method" course in finance in recent years will recall in this connection the famous Liquigas case of Hunt and Williams, [9, pp. 193–96] a case which is often used to introduce the student to the cost-of-capital problem and to poke a bit of fun at the economist's certainty-model.

[5] For an attempt at a rigorous explicit development of this line of attack, see F. Modigliani and M. Zeman [14].

to explore (within limits) some of the implications of different financing arrangements, and it does give some meaning to the "cost" of different types of funds. However, because the cost of capital has become an essentially subjective concept, the utility approach has serious drawbacks for normative as well as analytical purposes. How, for example, is management to ascertain the risk preferences of its stockholders and to compromise among their tastes? And how can the economist build a meaningful investment function in the face of the fact that any given investment opportunity might or might not be worth exploiting depending on precisely who happen to be the owners of the firm at the moment?

Fortunately, these questions do not have to be answered; for the alternative approach, based on market value maximization, can provide the basis for an operational definition of the cost of capital and a workable theory of investment. Under this approach any investment project and its concomitant financing plan must pass only the following test: Will the project, as financed, raise the market value of the firm's shares? If so, it is worth undertaking; if not, its return is less than the marginal cost of capital to the firm. Note that such a test is entirely independent of the tastes of the current owners, since market prices will reflect not only their preferences but those of all potential owners as well. If any current stockholder disagrees with management and the market over the valuation of the project, he is free to sell out and reinvest elsewhere, but will still benefit from the capital appreciation resulting from management's decision.

The potential advantages of the market-value approach have long been appreciated; yet analytical results have been meager. What appears to be keeping this line of development from achieving its promise is largely the lack of an adequate theory of the effect of financial structure on market valuations, and of how these effects can be inferred from objective market data. It is with the development of such a theory and of its implications for the cost-of-capital problem that we shall be concerned in this paper.

Our procedure will be to develop in Section I the basic theory itself and to give some brief account of its empirical relevance. In Section II, we show how the theory can be used to answer the cost-of-capital question and how it permits us to develop a theory of investment of the firm under conditions of uncertainty. Throughout these sections the approach is essentially a partial-equilibrium one focusing on the firm and "industry." Accordingly, the "prices" of certain income streams will be treated as constant and given from outside the model, just as in the standard Marshallian analysis of the firm and industry the prices of all inputs and of all other products are taken as given. We have chosen to focus at this level rather than on the economy as a whole because it is at the level of the firm and the industry that the interests of the various specialists concerned with the cost-of-capital problem come most closely together. Although the emphasis has thus been placed on partial-equilibrium analysis, the results obtained also provide the essential building blocks for a general equilibrium model which shows how those prices which are here taken as given, are themselves determined. For reasons of space, however, and because the material is of interest in its own right, the presentation of the general equilibrium model which rounds out the analysis must be deferred to a subsequent paper.

I. *The Valuation of Securities, Leverage, and the Cost of Capital*

A. *The Capitalization Rate for Uncertain Streams*

As a starting point, consider an economy in which all physical assets are owned by corporations. For the moment, assume that these corporations can finance their assets by issuing common stock only; the introduction of bond issues, or their equivalent, as a source of corporate funds is postponed until the next part of this section.

The physical assets held by each firm will yield to the owners of the firm—its stockholders—a stream of "profits" over time; but the elements of this series need not be constant and in any event are uncertain. This stream of income, and hence the stream accruing to any share of common stock, will be regarded as extending indefinitely into the future. We assume, however, that the mean value of the stream over time, or average profit per unit of time, is finite and represents a random variable subject to a (subjective) probability distribution. We shall refer to the average value over time of the stream accruing to a given share as the return of that share; and to the mathematical expectation of this average as the expected return of the share.[6] Although individual investors may have different views as to the shape of the probability distri-

[6] These propositions can be restated analytically as follows: The assets of the ith firm generate a stream:

$$X_i(1),\ X_i(2) \cdots X_i(T)$$

whose elements are random variables subject to the joint probability distribution:

$$\chi_i[X_i(1),\ X_i(2) \cdots X_i(t)].$$

The return to the ith firm is defined as:

$$X_i = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} X_i(t).$$

X_i is itself a random variable with a probability distribution $\Phi_i(X_i)$ whose form is determined uniquely by χ_i. The expected return $\bar{X}_i$ is defined as $\bar{X}_i = E(X_i) = \int_{X_i} X_i \Phi_i(X_i) dX_i$. If N_i is the number of shares outstanding, the return of the ith share is $x_i = (1/N)X_i$ with probability distribution $\phi_i(x_i)dx_i = \Phi_i(Nx_i)d(Nx_i)$ and expected value $\bar{x}_i = (1/N)\bar{X}_i$.

bution of the return of any share, we shall assume for simplicity that they are at least in agreement as to the expected return.[7]

This way of characterizing uncertain streams merits brief comment. Notice first that the stream is a stream of profits, not dividends. As will become clear later, as long as management is presumed to be acting in the best interests of the stockholders, retained earnings can be regarded as equivalent to a fully subscribed, pre-emptive issue of common stock. Hence, for present purposes, the division of the stream between cash dividends and retained earnings in any period is a mere detail. Notice also that the uncertainty attaches to the mean value over time of the stream of profits and should not be confused with variability over time of the successive elements of the stream. That variability and uncertainty are two totally different concepts should be clear from the fact that the elements of a stream can be variable even though known with certainty. It can be shown, furthermore, that whether the elements of a stream are sure or uncertain, the effect of variability per se on the valuation of the stream is at best a second-order one which can safely be neglected for our purposes (and indeed most others too).[8]

The next assumption plays a strategic role in the rest of the analysis. We shall assume that firms can be divided into "equivalent return" classes such that the return on the shares issued by any firm in any given class is proportional to (and hence perfectly correlated with) the return on the shares issued by any other firm in the same class. This assumption implies that the various shares within the same class differ, at most, by a "scale factor." Accordingly, if we adjust for the difference in scale, by taking the *ratio* of the return to the expected return, the probability distribution of that ratio is identical for all shares in the class. It follows that all relevant properties of a share are uniquely characterized by specifying (1) the class to which it belongs and (2) its expected return.

The significance of this assumption is that it permits us to classify firms into groups within which the shares of different firms are "homogeneous," that is, perfect substitutes for one another. We have, thus, an analogue to the familiar concept of the industry in which it is the commodity produced by the firms that is taken as homogeneous. To complete this analogy with Marshallian price theory, we shall assume in the

[7] To deal adequately with refinements such as differences among investors in estimates of expected returns would require extensive discussion of the theory of portfolio selection. Brief references to these and related topics will be made in the succeeding article on the general equilibrium model.

[8] The reader may convince himself of this by asking how much he would be willing to rebate to his employer for the privilege of receiving his annual salary in equal monthly installments rather than in irregular amounts over the year. See also J. M. Keynes [10, esp. pp. 53–54].

analysis to follow that the shares concerned are traded in perfect markets under conditions of atomistic competition.[9]

From our definition of homogeneous classes of stock it follows that in equilibrium in a perfect capital market the price per dollar's worth of expected return must be the same for all shares of any given class. Or, equivalently, in any given class the price of every share must be proportional to its expected return. Let us denote this factor of proportionality for any class, say the kth class, by $1/\rho_k$. Then if p_j denotes the price and $\bar{x}_j$ is the expected return per share of the jth firm in class k, we must have:

$$p_j = \frac{1}{\rho_k} \bar{x}_j; \tag{1}$$

or, equivalently,

$$\frac{\bar{x}_j}{p_j} = \rho_k \text{ a constant for all firms } j \text{ in class } k. \tag{2}$$

The constants ρ_k (one for each of the k classes) can be given several economic interpretations: (a) From (2) we see that each ρ_k is the expected rate of return of any share in class k. (b) From (1) $1/\rho_k$ is the price which an investor has to pay for a dollar's worth of expected return in the class k. (c) Again from (1), by analogy with the terminology for perpetual bonds, ρ_k can be regarded as the market rate of capitalization for the expected value of the uncertain streams of the kind generated by the kth class of firms.[10]

B. *Debt Financing and Its Effects on Security Prices*

Having developed an apparatus for dealing with uncertain streams we can now approach the heart of the cost-of-capital problem by dropping the assumption that firms cannot issue bonds. The introduction of debt-financing changes the market for shares in a very fundamental way. Because firms may have different proportions of debt in their capi-

[9] Just what our classes of stocks contain and how the different classes can be identified by outside observers are empirical questions to which we shall return later. For the present, it is sufficient to observe: (1) Our concept of a class, while not identical to that of the industry is at least closely related to it. Certainly the basic characteristics of the probability distributions of the returns on assets will depend to a significant extent on the product sold and the technology used. (2) What are the appropriate class boundaries will depend on the particular problem being studied. An economist concerned with general tendencies in the market, for example, might well be prepared to work with far wider classes than would be appropriate for an investor planning his portfolio, or a firm planning its financial strategy.

[10] We cannot, on the basis of the assumptions so far, make any statements about the relationship or spread between the various ρ's or capitalization rates. Before we could do so we would have to make further specific assumptions about the way investors believe the probability distributions vary from class to class, as well as assumptions about investors' preferences as between the characteristics of different distributions.

tal structure, shares of different companies, even in the same class, can give rise to different probability distributions of returns. In the language of finance, the shares will be subject to different degrees of financial risk or "leverage" and hence they will no longer be perfect substitutes for one another.

To exhibit the mechanism determining the relative prices of shares under these conditions, we make the following two assumptions about the nature of bonds and the bond market, though they are actually stronger than is necessary and will be relaxed later: (1) All bonds (including any debts issued by households for the purpose of carrying shares) are assumed to yield a constant income per unit of time, and this income is regarded as certain by all traders regardless of the issuer. (2) Bonds, like stocks, are traded in a perfect market, where the term perfect is to be taken in its usual sense as implying that any two commodities which are perfect substitutes for each other must sell, in equilibrium, at the same price. It follows from assumption (1) that all bonds are in fact perfect substitutes up to a scale factor. It follows from assumption (2) that they must all sell at the same price per dollar's worth of return, or what amounts to the same thing must yield the same rate of return. This rate of return will be denoted by r and referred to as the rate of interest or, equivalently, as the capitalization rate for sure streams. We now can derive the following two basic propositions with respect to the valuation of securities in companies with different capital structures:

Proposition I. Consider any company j and let $\overline{X}_j$ stand as before for the expected return on the assets owned by the company (that is, its expected profit before deduction of interest). Denote by D_j the market value of the debts of the company; by S_j the market value of its common shares; and by $V_j \equiv S_j + D_j$ the market value of all its securities or, as we shall say, the market value of the firm. Then, our Proposition I asserts that we must have in equilibrium:

$$V_j \equiv (S_j + D_j) = \overline{X}_j/\rho_k, \text{ for any firm } j \text{ in class } k. \tag{3}$$

That is, the *market value of any firm is independent of its capital structure and is given by capitalizing its expected return at the rate ρ_k appropriate to its class.*

This proposition can be stated in an equivalent way in terms of the firm's "average cost of capital," $\overline{X}_j/V_j$, which is the ratio of its expected return to the market value of all its securities. Our proposition then is:

$$\frac{\overline{X}_j}{(S_j + D_j)} \equiv \frac{\overline{X}_j}{V_j} = \rho_k, \text{ for any firm } j, \text{ in class } k. \tag{4}$$

That is, *the average cost of capital to any firm is completely independent of* *its capital structure and is equal to the capitalization rate of a pure equity stream of its class.*

To establish Proposition I we will show that as long as the relations (3) or (4) do not hold between any pair of firms in a class, arbitrage will take place and restore the stated equalities. We use the term arbitrage advisedly. For if Proposition I did not hold, an investor could buy and sell stocks and bonds in such a way as to exchange one income stream for another stream, identical in all relevant respects but selling at a lower price. The exchange would therefore be advantageous to the investor quite independently of his attitudes toward risk.[11] As investors exploit these arbitrage opportunities, the value of the overpriced shares will fall and that of the underpriced shares will rise, thereby tending to eliminate the discrepancy between the market values of the firms.

By way of proof, consider two firms in the same class and assume for simplicity only, that the expected return, $\bar{X}$, is the same for both firms. Let company 1 be financed entirely with common stock while company 2 has some debt in its capital structure. Suppose first the value of the levered firm, V_2, to be larger than that of the unlevered one, V_1. Consider an investor holding s_2 dollars' worth of the shares of company 2, representing a fraction α of the total outstanding stock, S_2. The return from this portfolio, denoted by Y_2, will be a fraction α of the income available for the stockholders of company 2, which is equal to the total return X_2 less the interest charge, rD_2. Since under our assumption of homogeneity, the anticipated total return of company 2, X_2, is, under all circumstances, the same as the anticipated total return to company 1, X_1, we can hereafter replace X_2 and X_1 by a common symbol X. Hence, the return from the initial portfolio can be written as:

$$Y_2 = \alpha(X - rD_2). \tag{5}$$

Now suppose the investor sold his αS_2 worth of company 2 shares and acquired instead an amount $s_1 = \alpha(S_2 + D_2)$ of the shares of company 1. He could do so by utilizing the amount αS_2 realized from the sale of his initial holding and borrowing an additional amount αD_2 on his own credit, pledging his new holdings in company 1 as a collateral. He would thus secure for himself a fraction $s_1/S_1 = \alpha(S_2 + D_2)/S_1$ of the shares and earnings of company 1. Making proper allowance for the interest payments on his personal debt αD_2, the return from the new portfolio, Y_1, is given by:

[11] In the language of the theory of choice, the exchanges are movements from inefficient points in the interior to efficient points on the boundary of the investor's opportunity set; and not movements between efficient points along the boundary. Hence for this part of the analysis nothing is involved in the way of specific assumptions about investor attitudes or behavior other than that investors behave consistently and prefer more income to less income, *ceteris paribus*.

$$(6) \qquad Y_1 = \frac{\alpha(S_2 + D_2)}{S_1} X - r\alpha D_2 = \alpha \frac{V_2}{V_1} X - r\alpha D_2.$$

Comparing (5) with (6) we see that as long as $V_2 > V_1$ we must have $Y_1 > Y_2$, so that it pays owners of company 2's shares to sell their holdings, thereby depressing S_2 and hence V_2; and to acquire shares of company 1, thereby raising S_1 and thus V_1. We conclude therefore that levered companies cannot command a premium over unlevered companies because investors have the opportunity of putting the equivalent leverage into their portfolio directly by borrowing on personal account.

Consider now the other possibility, namely that the market value of the levered company V_2 is less than V_1. Suppose an investor holds initially an amount s_1 of shares of company 1, representing a fraction α of the total outstanding stock, S_1. His return from this holding is:

$$Y_1 = \frac{s_1}{S_1} X = \alpha X.$$

Suppose he were to exchange this initial holding for another portfolio, also worth s_1, but consisting of s_2 dollars of stock of company 2 and of d dollars of bonds, where s_2 and d are given by:

$$(7) \qquad s_2 = \frac{S_2}{V_2} s_1, \qquad d = \frac{D_2}{V_2} s_1.$$

In other words the new portfolio is to consist of stock of company 2 and of bonds in the proportions S_2/V_2 and D_2/V_2, respectively. The return from the stock in the new portfolio will be a fraction s_2/S_2 of the total return to stockholders of company 2, which is $(X - rD_2)$, and the return from the bonds will be rd. Making use of (7), the total return from the portfolio, Y_2, can be expressed as follows:

$$Y_2 = \frac{s_2}{S_2}(X - rD_2) + rd = \frac{s_1}{V_2}(X - rD_2) + r\frac{D_2}{V_2}s_1 = \frac{s_1}{V_2}X = \alpha\frac{S_1}{V_2}X$$

(since $s_1 = \alpha S_1$). Comparing Y_2 with Y_1 we see that, if $V_2 < S_1 \equiv V_1$, then Y_2 will exceed Y_1. Hence it pays the holders of company 1's shares to sell these holdings and replace them with a mixed portfolio containing an appropriate fraction of the shares of company 2.

The acquisition of a mixed portfolio of stock of a levered company j and of bonds in the proportion S_j/V_j and D_j/V_j respectively, may be regarded as an operation which "undoes" the leverage, giving access to an appropriate fraction of the unlevered return X_j. It is this possibility of undoing leverage which prevents the value of levered firms from being consistently less than those of unlevered firms, or more generally prevents the average cost of capital $\overline{X}_j/V_j$ from being systematically higher for levered than for nonlevered companies in the same class.

Since we have already shown that arbitrage will also prevent V_2 from being larger than V_1, we can conclude that in equilibrium we must have $V_2 = V_1$, as stated in Proposition I.

Proposition II. From Proposition I we can derive the following proposition concerning the rate of return on common stock in companies whose capital structure includes some debt: the expected rate of return or yield, i, on the stock of any company j belonging to the kth class is a linear function of leverage as follows:

$$i_j = \rho_k + (\rho_k - r)D_j/S_j. \tag{8}$$

That is, *the expected yield of a share of stock is equal to the appropriate capitalization rate ρ_k for a pure equity stream in the class, plus a premium related to financial risk equal to the debt-to-equity ratio times the spread between ρ_k and r.* Or equivalently, the market price of any share of stock is given by capitalizing its expected return at the continuously variable rate i_j of (8).[12]

A number of writers have stated close equivalents of our Proposition I although by appealing to intuition rather than by attempting a proof and only to insist immediately that the results were not applicable to the actual capital markets.[13] Proposition II, however, so far as we have been able to discover is new.[14] To establish it we first note that, by definition, the expected rate of return, i, is given by:

$$i_j \equiv \frac{\bar{X}_j - rD_j}{S_j}. \tag{9}$$

From Proposition I, equation (3), we know that:

$$\bar{X}_j = \rho_k(S_j + D_j).$$

Substituting in (9) and simplifying, we obtain equation (8).

[12] To illustrate, suppose $\bar{X} = 1000$, $D = 4000$, $r = 5$ per cent and $\rho_k = 10$ per cent. These values imply that $V = 10{,}000$ and $S = 6000$ by virtue of Proposition I. The expected yield or rate of return per share is then:

$$i = \frac{1000 - 200}{6000} = .1 + (.1 - .05)\frac{4000}{6000} = 13\tfrac{1}{3} \text{ per cent.}$$

[13] See, for example, J. B. Williams [21, esp. pp. 72–73]; David Durand [3]; and W. A. Morton [15]. None of these writers describe in any detail the mechanism which is supposed to keep the average cost of capital constant under changes in capital structure. They seem, however, to be visualizing the equilibrating mechanism in terms of switches by investors between stocks and bonds as the yields of each get out of line with their "riskiness." This is an argument quite different from the pure arbitrage mechanism underlying our proof, and the difference is crucial. Regarding Proposition I as resting on investors' attitudes toward risk leads inevitably to a misunderstanding of many factors influencing relative yields such as, for example, limitations on the portfolio composition of financial institutions. See below, esp. Section I.D.

[14] Morton does make reference to a linear yield function but only " . . . for the sake of simplicity and because the particular function used makes no essential difference in my conclusions" [15, p. 443, note 2].

C. *Some Qualifications and Extensions of the Basic Propositions*

The methods and results developed so far can be extended in a number of useful directions, of which we shall consider here only three: (1) allowing for a corporate profits tax under which interest payments are deductible; (2) recognizing the existence of a multiplicity of bonds and interest rates; and (3) acknowledging the presence of market imperfections which might interfere with the process of arbitrage. The first two will be examined briefly in this section with some further attention given to the tax problem in Section II. Market imperfections will be discussed in Part D of this section in the course of a comparison of our results with those of received doctrines in the field of finance.

Effects of the Present Method of Taxing Corporations. The deduction of interest in computing taxable corporate profits will prevent the arbitrage process from making the value of all firms in a given class proportional to the expected returns generated by their physical assets. Instead, it can be shown (by the same type of proof used for the original version of Proposition I) that the market values of firms in each class must be proportional in equilibrium to their expected return net of taxes (that is, to the sum of the interest paid and expected net stockholder income). This means we must replace each $\bar{X}_j$ in the original versions of Propositions I and II with a new variable $\bar{X}_j^\tau$ representing the total income net of taxes generated by the firm:

$$\bar{X}_j^\tau \equiv (\bar{X}_j - rD_j)(1 - \tau) + rD_j \equiv \bar{\pi}_j^\tau + rD_j, \tag{10}$$

where $\bar{\pi}_j^\tau$ represents the expected net income accruing to the common stockholders and τ stands for the average rate of corporate income tax.[15]

After making these substitutions, the propositions, when adjusted for taxes, continue to have the same form as their originals. That is, Proposition I becomes:

$$\frac{\bar{X}_j^\tau}{V_j} = \rho_k^\tau, \text{ for any firm in class } k, \tag{11}$$

and Proposition II becomes

$$i_j \equiv \frac{\bar{\pi}_j^\tau}{S_j} = \rho_j^\tau + (\rho_k^\tau - r) D_j/S_j \tag{12}$$

where ρ_k^τ is the capitalization rate for income net of taxes in class k.

Although the form of the propositions is unaffected, certain interpretations must be changed. In particular, the after-tax capitalization rate

[15] For simplicity, we shall ignore throughout the tiny element of progression in our present corporate tax and treat τ as a constant independent of $(X_j - rD_j)$.

ρ_k^τ can no longer be identified with the "average cost of capital" which is $\rho_k = \overline{X}_j / V_j$. The difference between ρ_k^τ and the "true" average cost of capital, as we shall see, is a matter of some relevance in connection with investment planning within the firm (Section II). For the description of market behavior, however, which is our immediate concern here, the distinction is not essential. To simplify presentation, therefore, and to preserve continuity with the terminology in the standard literature we shall continue in this section to refer to ρ_k^τ as the average cost of capital, though strictly speaking this identification is correct only in the absence of taxes.

Effects of a Plurality of Bonds and Interest Rates. In existing capital markets we find not one, but a whole family of interest rates varying with maturity, with the technical provisions of the loan and, what is most relevant for present purposes, with the financial condition of the borrower.[16] Economic theory and market experience both suggest that the yields demanded by lenders tend to increase with the debt-equity ratio of the borrowing firm (or individual). If so, and if we can assume as a first approximation that this yield curve, $r = r\ (D/S)$, whatever its precise form, is the same for all borrowers, then we can readily extend our propositions to the case of a rising supply curve for borrowed funds.[17]

Proposition I is actually unaffected in form and interpretation by the fact that the rate of interest may rise with leverage; while the average cost of *borrowed* funds will tend to increase as debt rises, the average cost of funds from *all* sources will still be independent of leverage (apart from the tax effect). This conclusion follows directly from the ability of those who engage in arbitrage to undo the leverage in any financial structure by acquiring an appropriately mixed portfolio of bonds and stocks. Because of this ability, the ratio of earnings (*before* interest charges) to market value—*i.e.*, the average cost of capital from all

[16] We shall not consider here the extension of the analysis to encompass the time structure of interest rates. Although some of the problems posed by the time structure can be handled within our comparative statics framework, an adequate discussion would require a separate paper.

[17] We can also develop a theory of bond valuation along lines essentially parallel to those followed for the case of shares. We conjecture that the curve of bond yields as a function of leverage will turn out to be a nonlinear one in contrast to the linear function of leverage developed for common shares. However, we would also expect that the rate of increase in the yield on new issues would not be substantial in practice. This relatively slow rise would reflect the fact that interest rate increases by themselves can never be completely satisfactory to creditors as compensation for their increased risk. Such increases may simply serve to raise r so high relative to ρ that they become self-defeating by giving rise to a situation in which even normal fluctuations in earnings may force the company into bankruptcy. The difficulty of borrowing more, therefore, tends to show up in the usual case not so much in higher rates as in the form of increasingly stringent restrictions imposed on the company's management and finances by the creditors; and ultimately in a complete inability to obtain new borrowed funds, at least from the institutional investors who normally set the standards in the market for bonds.

sources—must be the same for all firms in a given class.[18] In other words, the increased cost of borrowed funds as leverage increases will tend to be offset by a corresponding reduction in the yield of common stock. This seemingly paradoxical result will be examined more closely below in connection with Proposition II.

A significant modification of Proposition I would be required only if the yield curve $r=r(D/S)$ were different for different borrowers, as might happen if creditors had marked preferences for the securities of a particular class of debtors. If, for example, corporations as a class were able to borrow at lower rates than individuals having equivalent personal leverage, then the average cost of capital to corporations might fall slightly, as leverage increased over some range, in reflection of this differential. In evaluating this possibility, however, remember that the relevant interest rate for our arbitrage operators is the rate on brokers' loans and, historically, that rate has not been noticeably higher than representative corporate rates.[19] The operations of holding companies and investment trusts which can borrow on terms comparable to operating companies represent still another force which could be expected to wipe out any marked or prolonged advantages from holding levered stocks.[20]

Although Proposition I remains unaffected as long as the yield curve is the same for all borrowers, the relation between common stock yields and leverage will no longer be the strictly linear one given by the original Proposition II. If r increases with leverage, the yield i will still tend to

[18] One normally minor qualification might be noted. Once we relax the assumption that all bonds have certain yields, our arbitrage operator faces the danger of something comparable to "gambler's ruin." That is, there is always the possibility that an otherwise sound concern—one whose long-run expected income is greater than its interest liability—might be forced into liquidation as a result of a run of temporary losses. Since reorganization generally involves costs, and because the operation of the firm may be hampered during the period of reorganization with lasting unfavorable effects on earnings prospects, we might perhaps expect heavily levered companies to sell at a slight discount relative to less heavily indebted companies of the same class.

[19] Under normal conditions, moreover, a substantial part of the arbitrage process could be expected to take the form, not of having the arbitrage operators go into debt on personal account to put the required leverage into their portfolios, but simply of having them reduce the amount of corporate bonds they already hold when they acquire underpriced unlevered stock. Margin requirements are also somewhat less of an obstacle to maintaining any desired degree of leverage in a portfolio than might be thought at first glance. Leverage could be largely restored in the face of higher margin requirements by switching to stocks having more leverage at the corporate level.

[20] An extreme form of inequality between borrowing and lending rates occurs, of course, in the case of preferred stocks, which can not be directly issued by individuals on personal account. Here again, however, we would expect that the operations of investment corporations plus the ability of arbitrage operators to sell off their holdings of preferred stocks would act to prevent the emergence of any substantial premiums (for this reason) on capital structures containing preferred stocks. Nor are preferred stocks so far removed from bonds as to make it impossible for arbitrage operators to approximate closely the risk and leverage of a corporate preferred stock by incurring a somewhat smaller debt on personal account.

rise as D/S increases, but at a decreasing rather than a constant rate. Beyond some high level of leverage, depending on the exact form of the interest function, the yield may even start to fall.[21] The relation between i and D/S could conceivably take the form indicated by the curve MD

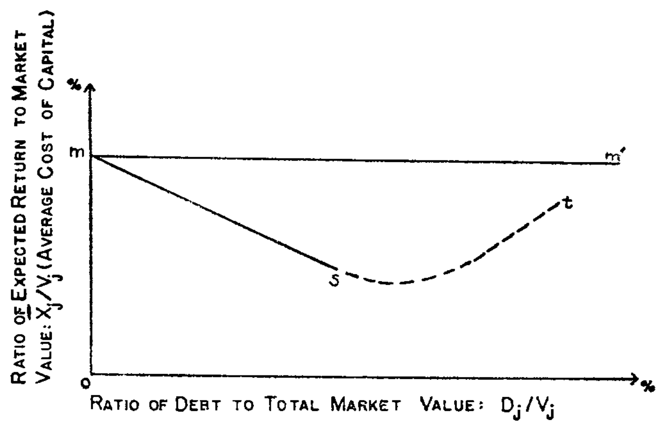

FIGURE 1

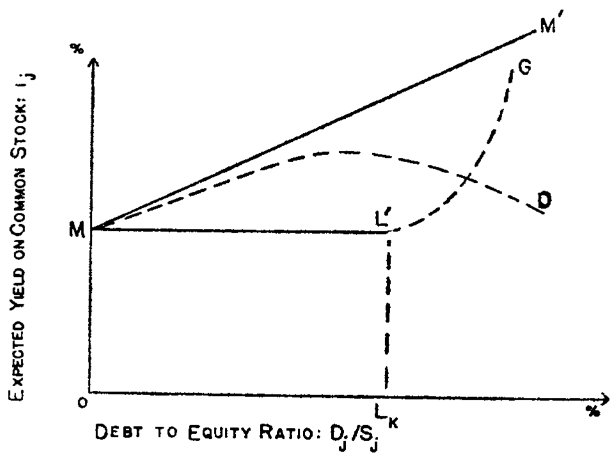

FIGURE 2

in Figure 2, although in practice the curvature would be much less pronounced. By contrast, with a constant rate of interest, the relation would be linear throughout as shown by line MM', Figure 2.

The downward sloping part of the curve MD perhaps requires some

[21] Since new lenders are unlikely to permit this much leverage (*cf.* note 17), this range of the curve is likely to be occupied by companies whose earnings prospects have fallen substantially since the time when their debts were issued.

comment since it may be hard to imagine why investors, other than those who like lotteries, would purchase stocks in this range. Remember, however, that the yield curve of Proposition II is a consequence of the more fundamental Proposition I. Should the demand by the risk-lovers prove insufficient to keep the market to the peculiar yield-curve MD, this demand would be reinforced by the action of arbitrage operators. The latter would find it profitable to own a pro-rata share of the firm as a whole by holding its stock *and* bonds, the lower yield of the shares being thus offset by the higher return on bonds.

D. *The Relation of Propositions I and II to Current Doctrines*

The propositions we have developed with respect to the valuation of firms and shares appear to be substantially at variance with current doctrines in the field of finance. The main differences between our view and the current view are summarized graphically in Figures 1 and 2. Our Proposition I [equation (4)] asserts that the average cost of capital, $\bar{X}_j^\tau/V_j$, is a constant for all firms j in class k, independently of their financial structure. This implies that, if we were to take a sample of firms in a given class, and if for each firm we were to plot the ratio of expected return to market value against some measure of leverage or financial structure, the points would tend to fall on a horizontal straight line with intercept ρ_k^τ, like the solid line mm' in Figure 1.[22] From Proposition I we derived Proposition II [equation (8)] which, taking the simplest version with r constant, asserts that, for all firms in a class, the relation between the yield on common stock and financial structure, measured by D_j/S_j, will approximate a straight line with slope $(\rho_k^\tau - r)$ and intercept ρ_k^τ. This relationship is shown as the solid line MM' in Figure 2, to which reference has been made earlier.[23]

By contrast, the conventional view among finance specialists appears to start from the proposition that, other things equal, the earnings-price ratio (or its reciprocal, the times-earnings multiplier) of a firm's common stock will normally be only slightly affected by "moderate" amounts of debt in the firm's capital structure.[24] Translated into our no-

[22] In Figure 1 the measure of leverage used is D_j/V_j (the ratio of debt to market value) rather than D_j/S_j (the ratio of debt to equity), the concept used in the analytical development. The D_j/V_j measure is introduced at this point because it simplifies comparison and contrast of our view with the traditional position.

[23] The line MM' in Figure 2 has been drawn with a positive slope on the assumption that $\rho_k^\tau > r$, a condition which will normally obtain. Our Proposition II as given in equation (8) would continue to be valid, of course, even in the unlikely event that $\rho_k^\tau < r$, but the slope of MM' would be negative.

[24] See, *e.g.*, Graham and Dodd [6, pp. 464–66]. Without doing violence to this position, we can bring out its implications more sharply by ignoring the qualification and treating the yield as a virtual constant over the relevant range. See in this connection the discussion in Durand [3, esp. pp. 225–37] of what he calls the "net income method" of valuation.

tation, it asserts that for any firm j in the class k,

$$\frac{\overline{X}_j^\tau - rD_j}{S_j} \equiv \frac{\bar{\pi}_j^\tau}{S_j} = i_k^*, \text{ a constant for } \frac{D_j}{S_j} \leq L_k \tag{13}$$

or, equivalently,

$$S_j = \bar{\pi}_j^\tau / i_k^*. \tag{14}$$

Here i_k^* represents the capitalization rate or earnings-price ratio on the common stock and L_k denotes some amount of leverage regarded as the maximum "reasonable" amount for firms of the class k. This assumed relationship between yield and leverage is the horizontal solid line ML' of Figure 2. Beyond L', the yield will presumably rise sharply as the market discounts "excessive" trading on the equity. This possibility of a rising range for high leverages is indicated by the broken-line segment $L'G$ in the figure.[25]

If the value of shares were really given by (14) then the over-all market value of the firm must be:

$$V_j \equiv S_j + D_j = \frac{\overline{X}_j^\tau - rD_j}{i_k^*} + D_j = \frac{\overline{X}_j^\tau}{i_k^*} + \frac{(i_k^* - r)D_j}{i_k^*}. \tag{16}$$

That is, for any given level of expected total returns after taxes ($\overline{X}_j^\tau$) and assuming, as seems natural, that $i_k^* > r$, the value of the firm must tend to *rise* with debt;[26] whereas our Proposition I asserts that the value of the firm is completely independent of the capital structure. Another way of contrasting our position with the traditional one is in terms of the cost of capital. Solving (16) for $\overline{X}_j^\tau / V_j$ yields:

$$\overline{X}_j^\tau / V_j = i_k^* - (i_k^* - r)D_j/V_j. \tag{17}$$

According to this equation, the average cost of capital is not independent of capital structure as we have argued, but should tend to *fall* with increasing leverage, at least within the relevant range of moderate debt ratios, as shown by the line *ms* in Figure 1. Or to put it in more familiar terms, debt-financing should be "cheaper" than equity-financing if not carried too far.

When we also allow for the possibility of a rising range of stock yields for large values of leverage, we obtain a U-shaped curve like *mst* in

[25] To make it easier to see some of the implications of this hypothesis as well as to prepare the ground for later statistical testing, it will be helpful to assume that the notion of a critical limit on leverage beyond which yields rise rapidly, can be epitomized by a quadratic relation of the form:

$$\bar{\pi}_j^\tau / S_j = i_k^* + \beta(D_j/S_j) + \alpha(D_j/S_j)^2, \qquad \alpha > 0. \tag{15}$$

[26] For a typical discussion of how a promoter can, supposedly, increase the market value of a firm by recourse to debt issues, see W. J. Eiteman [4, esp. pp. 11–13].

Figure 1.[27] That a yield-curve for stocks of the form $ML'G$ in Figure 2 implies a U-shaped cost-of-capital curve has, of course, been recognized by many writers. A natural further step has been to suggest that the capital structure corresponding to the trough of the U is an "optimal capital structure" towards which management ought to strive in the best interests of the stockholders.[28] According to our model, by contrast, no such optimal structure exists—all structures being equivalent from the point of view of the cost of capital.

Although the falling, or at least U-shaped, cost-of-capital function is in one form or another the dominant view in the literature, the ultimate rationale of that view is by no means clear. The crucial element in the position—that the expected earnings-price ratio of the stock is largely unaffected by leverage up to some conventional limit—is rarely even regarded as something which requires explanation. It is usually simply taken for granted or it is merely asserted that this is the way the market behaves.[29] To the extent that the constant earnings-price ratio has a rationale at all we suspect that it reflects in most cases the feeling that moderate amounts of debt in "sound" corporations do not really add very much to the "riskiness" of the stock. Since the extra risk is slight, it seems natural to suppose that firms will not have to pay noticeably higher yields in order to induce investors to hold the stock.[30]

A more sophisticated line of argument has been advanced by David Durand [3, pp. 231-33]. He suggests that because insurance companies and certain other important institutional investors are restricted to debt securities, nonfinancial corporations are able to borrow from them at interest rates which are lower than would be required to compensate

[27] The U-shaped nature of the cost-of-capital curve can be exhibited explicitly if the yield curve for shares as a function of leverage can be approximated by equation (15) of footnote 25. From that equation, multiplying both sides by S_j we obtain: $\bar{\pi}_j^\tau = \bar{X}_j^\tau - rD_j = i_k^* S_j + \beta D_j + \alpha D_j^2 / S_j$ or, adding and subtracting $i_k^* D_k$ from the right-hand side and collecting terms,

$$\bar{X}_j^\tau = i_k^*(S_j + D_j) + (\beta + r - i_k^*)D_j + \alpha D^2{}_j/S_j. \tag{18}$$

Dividing (18) by V_j gives an expression for the cost of capital:

$$\begin{aligned}\bar{X}_j^\tau / V_j = i_k^* - (i_k^* - r - \beta)D_j/V_j + \alpha D_j^2/S_jV_j = i_k^* - (i_k^* - r - \beta)D_j/V_j \\ + \alpha(D_j/V_j)^2/(1 - D_j/V_j)\end{aligned} \tag{19}$$

which is clearly U-shaped since α is supposed to be positive.

[28] For a typical statement see S. M. Robbins [16, p. 307]. See also Graham and Dodd [6, pp. 468–74].

[29] See *e.g.*, Graham and Dodd [6, p. 466].

[30] A typical statement is the following by Guthmann and Dougall [7, p. 245]: "Theoretically it might be argued that the increased hazard from using bonds and preferred stocks would counterbalance this additional income and so prevent the common stock from being more attractive than when it had a lower return but fewer prior obligations. In practice, the extra earnings from 'trading on the equity' are often regarded by investors as more than sufficient to serve as a 'premium for risk' when the proportions of the several securities are judiciously mixed."

creditors in a free market. Thus, while he would presumably agree with our conclusions that stockholders could not gain from leverage in an unconstrained market, he concludes that they can gain under present institutional arrangements. This gain would arise by virtue of the "safety superpremium" which lenders are willing to pay corporations for the privilege of lending.[31]

The defective link in both the traditional and the Durand version of the argument lies in the confusion between investors' subjective risk preferences and their objective market opportunities. Our Propositions I and II, as noted earlier, do not depend for their validity on any assumption about individual risk preferences. Nor do they involve any assertion as to what is an adequate compensation to investors for assuming a given degree of risk. They rely merely on the fact that a given commodity cannot consistently sell at more than one price in the market; or more precisely that the price of a commodity representing a "bundle" of two other commodities cannot be consistently different from the weighted average of the prices of the two components (the weights being equal to the proportion of the two commodities in the bundle).

An analogy may he helpful at this point. The relations between $1/\rho_k$, the price per dollar of an unlevered stream in class k; $1/r$, the price per dollar of a sure stream, and $1/i_j$, the price per dollar of a levered stream j, in the kth class, are essentially the same as those between, respectively, the price of whole milk, the price of butter fat, and the price of milk which has been thinned out by skimming off some of the butter fat. Our Proposition I states that a firm cannot reduce the cost of capital—*i.e.*, increase the market value of the stream it generates—by securing part of its capital through the sale of bonds, even though debt money appears to be cheaper. This assertion is equivalent to the proposition that, under perfect markets, a dairy farmer cannot in general earn more for the milk he produces by skimming some of the butter fat and selling it separately, even though butter fat per unit weight, sells for more than whole milk. The advantage from skimming the milk rather than selling whole milk would be purely illusory; for what would be gained from selling the high-priced butter fat would be lost in selling the low-priced residue of thinned milk. Similarly our Proposition II—that the price per dollar of a levered stream falls as leverage increases—is an ex-

[31] Like Durand, Morton [15] contends "that the actual market deviates from [Proposition I] by giving a changing over-all cost of money at different points of the [leverage] scale" (p. 443, note 2, inserts ours), but the basis for this contention is nowhere clearly stated. Judging by the great emphasis given to the lack of mobility of investment funds between stocks and bonds and to the psychological and institutional pressures toward debt portfolios (see pp. 444–51 and especially his discussion of the optimal capital structure on p. 453) he would seem to be taking a position very similar to that of Durand above.

act analogue of the statement that the price per gallon of thinned milk falls continuously as more butter fat is skimmed off.[32]

It is clear that this last assertion is true as long as butter fat is worth more per unit weight than whole milk, and it holds even if, for many consumers, taking a little cream out of the milk (adding a little leverage to the stock) does not detract noticeably from the taste (does not add noticeably to the risk). Furthermore the argument remains valid even in the face of instituional limitations of the type envisaged by Durand. For suppose that a large fraction of the population habitually dines in restaurants which are required by law to serve only cream in lieu of milk (entrust their savings to institutional investors who can only buy bonds). To be sure the price of butter fat will then tend to be higher in relation to that of skimmed milk than in the absence such restrictions (the rate of interest will tend to be lower), and this will benefit people who eat at home and who like skim milk (who manage their own portfolio and are able and willing to take risk). But it will still be the case that a farmer cannot gain by skimming some of the butter fat and selling it separately (firm cannot reduce the cost of capital by recourse to borrowed funds).[33]

Our propositions can be regarded as the extension of the classical theory of markets to the particular case of the capital markets. Those who hold the current view—whether they realize it or not—must as-

[32] Let M denote the quantity of whole milk, B/M the proportion of butter fat in the whole milk, and let p_M, p_B and p_α denote, respectively, the price per unit weight of whole milk, butter fat and thinned milk from which a fraction α of the butter fat has been skimmed off. We then have the fundamental perfect market relation:

$$p_\alpha(M - \alpha B) + p_B \alpha B = p_M M, \qquad 0 \leq \alpha \leq 1, \tag{a}$$

stating that total receipts will be the same amount $p_M M$, independently of the amount αB of butter fat that may have been sold separately. Since p_M corresponds to $1/\rho$, p_B to $1/r$, p_α to $1/i$, M to $\bar{X}$ and αB to rD, (a) is equivalent to Proposition I, $S+D=\bar{X}/\rho$. From (a) we derive:

$$p_\alpha = p_M \frac{M}{M - \alpha B} - p_B \frac{\alpha B}{M - \alpha B} \tag{b}$$

which gives the price of thinned milk as an explicit function of the proportion of butter fat skimmed off; the function decreasing as long as $p_B > p_M$. From (a) also follows:

$$1/p_\alpha = 1/p_M + (1/p_M - 1/p_B) \frac{p_B \alpha B}{p_\alpha(M - \alpha B)} \tag{c}$$

which is the exact analogue of Proposition II, as given by (8).

[33] The reader who likes parables will find that the analogy with interrelated commodity markets can be pushed a good deal farther than we have done in the text. For instance, the effect of changes in the market rate of interest on the over-all cost of capital is the same as the effect of a change in the price of butter on the price of whole milk. Similarly, just as the relation between the prices of skim milk and butter fat influences the kind of cows that will be reared, so the relation between i and r influences the kind of ventures that will be undertaken. If people like butter we shall have Guernseys; if they are willing to pay a high price for safety, this will encourage ventures which promise smaller but less uncertain streams per dollar of physical assets.

sume not merely that there are lags and frictions in the equilibrating process—a feeling we certainly share,[34] claiming for our propositions only that they describe the central tendency around which observations will scatter—but also that there are large and *systematic* imperfections in the market which permanently bias the outcome. This is an assumption that economists, at any rate, will instinctively eye with some skepticism.

In any event, whether such prolonged, systematic departures from equilibrium really exist or whether our propositions are better descriptions of long-run market behavior can be settled only by empirical research. Before going on to the theory of investment it may be helpful, therefore, to look at the evidence.

E. *Some Preliminary Evidence on the Basic Propositions*

Unfortunately the evidence which has been assembled so far is amazingly skimpy. Indeed, we have been able to locate only two recent studies—and these of rather limited scope—which were designed to throw light on the issue. Pending the results of more comprehensive tests which we hope will soon be available, we shall review briefly such evidence as is provided by the two studies in question: (1) an analysis of the relation between security yields and financial structure for some 43 large electric utilities by F. B. Allen [1], and (2) a parallel (unpublished) study by Robert Smith [19], for 42 oil companies designed to test whether Allen's rather striking results would be found in an industry with very different characteristics.[35] The Allen study is based on average figures for the years 1947 and 1948, while the Smith study relates to the single year 1953.

The Effect of Leverage on the Cost of Capital. According to the received view, as shown in equation (17) the average cost of capital, $\overline{X}^{\tau}/V$, should decline linearly with leverage as measured by the ratio D/V, at least through most of the relevant range.[36] According to Proposition I, the average cost of capital within a given class k should tend to have the same value ρ_k^{τ} independently of the degree of leverage. A simple test

[34] Several specific examples of the failure of the arbitrage mechanism can be found in Graham and Dodd [6, *e.g.*, pp. 646–48]. The price discrepancy described on pp. 646–47 is particularly curious since it persists even today despite the fact that a whole generation of security analysts has been brought up on this book!

[35] We wish to express our thanks to both writers for making available to us some of their original worksheets. In addition to these recent studies there is a frequently cited (but apparently seldom read) study by the Federal Communications Commission in 1938 [22] which purports to show the existence of an optimal capital structure or range of structures (in the sense defined above) for public utilities in the 1930's. By current standards for statistical investigations, however, this study cannot be regarded as having any real evidential value for the problem at hand.

[36] We shall simplify our notation in this section by dropping the subscript j used to denote a particular firm wherever this will not lead to confusion.

of the merits of the two alternative hypotheses can thus be carried out by correlating $\bar{X}^{\tau}/V$ with D/V. If the traditional view is correct, the correlation should be significantly negative; if our view represents a better approximation to reality, then the correlation should not be significantly different from zero.

Both studies provide information about the average value of D—the market value of bonds and preferred stock—and of V—the market value of all securities.[37] From these data we can readily compute the ratio D/V and this ratio (expressed as a percentage) is represented by the symbol d in the regression equations below. The measurement of the variable $\bar{X}^{\tau}/V$, however, presents serious difficulties. Strictly speaking, the numerator should measure the expected returns net of taxes, but this is a variable on which no direct information is available. As an approximation, we have followed both authors and used (1) the average value of actual net returns in 1947 and 1948 for Allen's utilities; and (2) actual net returns in 1953 for Smith's oil companies. Net return is defined in both cases as the sum of interest, preferred dividends and stockholders' income net of corporate income taxes. Although this approximation to expected returns is undoubtedly very crude, there is no reason to believe that it will systematically bias the test in so far as the sign of the regression coefficient is concerned. The roughness of the approximation, however, will tend to make for a wide scatter. Also contributing to the scatter is the crudeness of the industrial classification, since especially within the sample of oil companies, the assumption that all the firms belong to the same class in our sense, is at best only approximately valid.

Denoting by x our approximation to $\bar{X}^{\tau}/V$ (expressed, like d, as a percentage), the results of the tests are as follows:

$$\text{Electric Utilities} \quad x = 5.3 + \underset{(\pm .008)}{.006d} \qquad r = .12$$

$$\text{Oil Companies} \quad x = 8.5 + \underset{(\pm .024)}{.006d} \qquad r = .04.$$

The data underlying these equations are also shown in scatter diagram form in Figures 3 and 4.

The results of these tests are clearly favorable to our hypothesis.

[37] Note that for purposes of this test preferred stocks, since they represent an *expected* fixed obligation, are properly classified with bonds even though the tax status of preferred dividends is different from that of interest payments and even though preferred dividends are really fixed only as to their maximum in any year. Some difficulty of classification does arise in the case of convertible preferred stocks (and convertible bonds) selling at a substantial premium, but fortunately very few such issues were involved for the companies included in the two studies. Smith included bank loans and certain other short-term obligations (at book values) in his data on oil company debts and this treatment is perhaps open to some question. However, the amounts involved were relatively small and check computations showed that their elimination would lead to only minor differences in the test results.

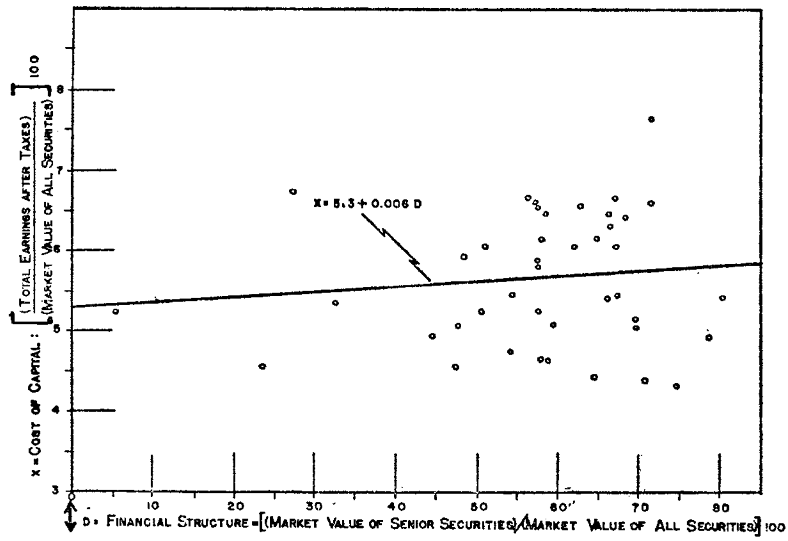

FIGURE 3. COST OF CAPITAL IN RELATION TO FINANCIAL STRUCTURE FOR 43 ELECTRIC UTILITIES, 1947–48

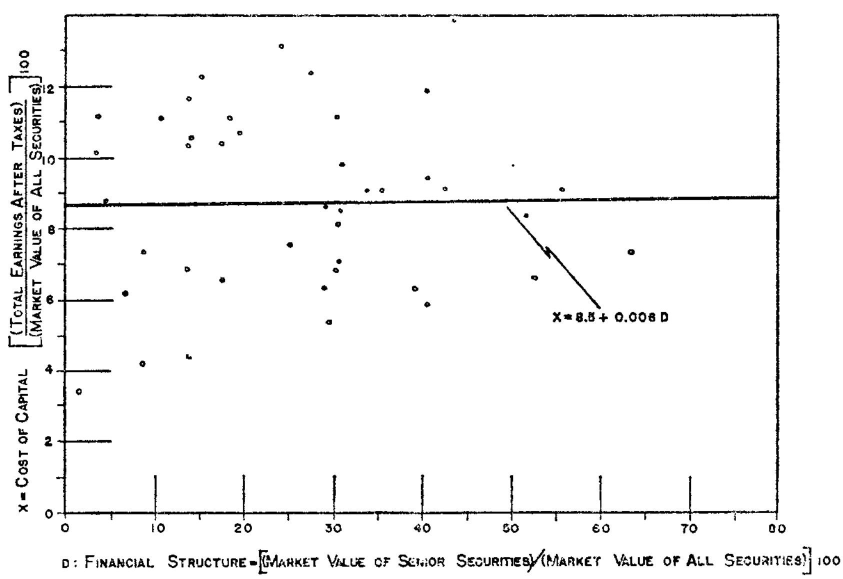

FIGURE 4. COST OF CAPITAL IN RELATION TO FINANCIAL STRUCTURE FOR 42 OIL COMPANIES, 1953

Both correlation coefficients are very close to zero and not statistically significant. Furthermore, the implications of the traditional view fail to be supported even with respect to the sign of the correlation. The data in short provide no evidence of any tendency for the cost of capital to fall as the debt ratio increases.[38]

It should also be apparent from the scatter diagrams that there is no hint of a curvilinear, U-shaped, relation of the kind which is widely believed to hold between the cost of capital and leverage. This graphical impression was confirmed by statistical tests which showed that for both industries the curvature was not significantly different from zero, its sign actually being opposite to that hypothesized.[39]

Note also that according to our model, the constant terms of the regression equations are measures of ρ_k^τ, the capitalization rates for unlevered streams and hence the average cost of capital in the classes in question. The estimates of 8.5 per cent for the oil companies as against 5.3 per cent for electric utilities appear to accord well with a priori expectations, both in absolute value and relative spread.

The Effect of Leverage on Common Stock Yields. According to our Proposition II—see equation 12 and Figure 2—the expected yield on common stock, $\bar{\pi}^\tau/S$, in any given class, should tend to increase with leverage as measured by the ratio D/S. The relation should tend to be linear and with positive slope through most of the relevant range (as in the curve MM' of Figure 2), though it might tend to flatten out if we move

[38] It may be argued that a test of the kind used is biased against the traditional view. The fact that both sides of the regression equation are divided by the variable V which may be subject to random variation might tend to impart a positive bias to the correlation. As a check on the results presented in the text, we have, therefore, carried out a supplementary test based on equation (16). This equation shows that, if the traditional view is correct, the market value of a company should, for given $\bar{X}^\tau$, increase with debt through most of the relevant range; according to our model the market value should be uncorrelated with D, given $\bar{X}^\tau$. Because of wide variations in the size of the firms included in our samples, all variables must be divided by a suitable scale factor in order to avoid spurious results in carrying out a test of equation (16). The factor we have used is the book value of the firm denoted by A. The hypothesis tested thus takes the specific form:

$$V/A = a + b(\bar{X}^\tau/A) + c(D/A)$$

and the numerator of the ratio X^τ/A is again approximated by actual net returns. The partial correlation between V/A and D/A should now be positive according to the traditional view and zero according to our model. Although division by A should, if anything, bias the results in favor of the traditional hypothesis, the partial correlation turns out to be only .03 for the oil companies and $-.28$ for the electric utilities. Neither of these coefficients is significantly different from zero and the larger one even has the wrong sign.

[39] The tests consisted of fitting to the data the equation (19) of footnote 27. As shown there, it follows from the U-shaped hypothesis that the coefficient α of the variable $(D/V)^2/(1-D/V)$, denoted hereafter by d^*, should be significant and positive. The following regression equations and partials were obtained:

Electric Utilities $x = 5.0 + .017d - .003d^*;\ r_{xd^*.d} = -.15$

Oil Companies $x = 8.0 + .05d - .03d^*;\ r_{xd^*.d} = -.14.$

far enough to the right (as in the curve MD'), to the extent that high leverage tends to drive up the cost of senior capital. According to the conventional view, the yield curve as a function of leverage should be a horizontal straight line (like ML') through most of the relevant range; far enough to the right, the yield may tend to rise at an increasing rate. Here again, a straight-forward correlation—in this case between $\bar{\pi}^\tau/S$ and D/S—can provide a test of the two positions. If our view is correct, the correlation should be significantly positive; if the traditional view is correct, the correlation should be negligible.

Subject to the same qualifications noted above in connection with $\bar{X}^\tau$, we can approximate $\bar{\pi}^\tau$ by actual stockholder net income.[40] Letting z denote in each case the approximation to $\bar{\pi}^\tau/S$ (expressed as a percentage) and letting h denote the ratio D/S (also in percentage terms) the following results are obtained:

$$\text{Electric Utilities} \quad z = 6.6 + \underset{(+.004)}{.017h} \qquad r = .53$$

$$\text{Oil Companies} \quad z = 8.9 + \underset{(\pm .012)}{.051h} \qquad r = .53.$$

These results are shown in scatter diagram form in Figures 5 and 6.

Here again the implications of our analysis seem to be borne out by the data. Both correlation coefficients are positive and highly significant when account is taken of the substantial sample size. Furthermore, the estimates of the coefficients of the equations seem to accord reasonably well with our hypothesis. According to equation (12) the constant term should be the value of ρ_k^τ for the given class while the slope should be $(\rho_k^\tau - r)$. From the test of Proposition I we have seen that for the oil companies the mean value of ρ_k^τ could be estimated at around 8.7. Since the average yield of senior capital during the period covered was in the order of $3\frac{1}{2}$ per cent, we should expect a constant term of about 8.7 per cent and a slope of just over 5 per cent. These values closely approximate the regression estimates of 8.9 per cent and 5.1 per cent respectively. For the electric utilities, the yield of senior capital was also on the order of $3\frac{1}{2}$ per cent during the test years, but since the estimate of the mean value of ρ_k^τ from the test of Proposition I was 5.6 per cent,

[40] As indicated earlier, Smith's data were for the single year 1953. Since the use of a single year's profits as a measure of expected profits might be open to objection we collected profit data for 1952 for the same companies and based the computation of $\bar{\pi}^\tau/S$ on the average of the two years. The value of $\bar{\pi}^\tau/S$ was obtained from the formula:

$$\left(\text{net earnings in } 1952 \cdot \frac{\text{assets in '53}}{\text{assets in '52}} + \text{net earnings in '1953}\right)\frac{1}{2}$$
$$\div \text{ (average market value of common stock in '53).}$$

The asset adjustment was introduced as rough allowance for the effects of possible growth in the size of the firm. It might be added that the correlation computed with $\bar{\pi}^\tau/S$ based on net profits in 1953 alone was found to be only slightly smaller, namely .50.

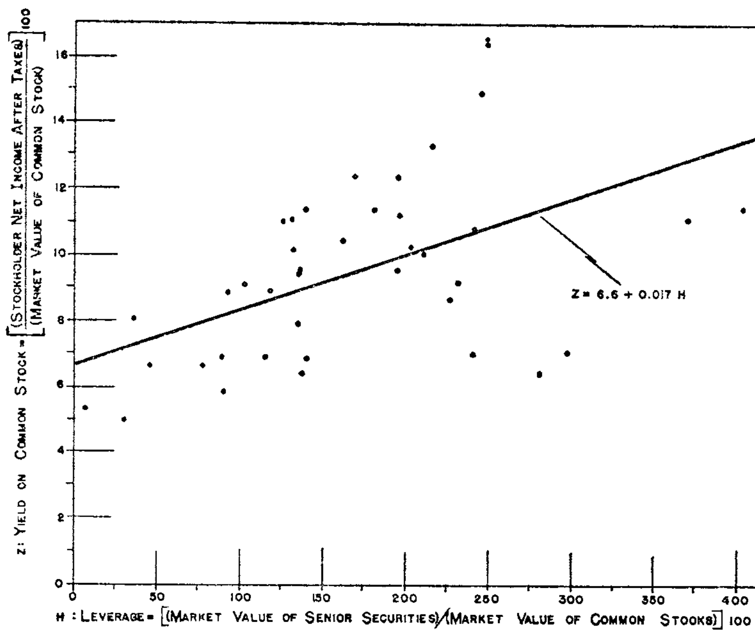

FIGURE 5. YIELD ON COMMON STOCK IN RELATION TO LEVERAGE FOR 43 ELECTRIC UTILITIES, 1947–48

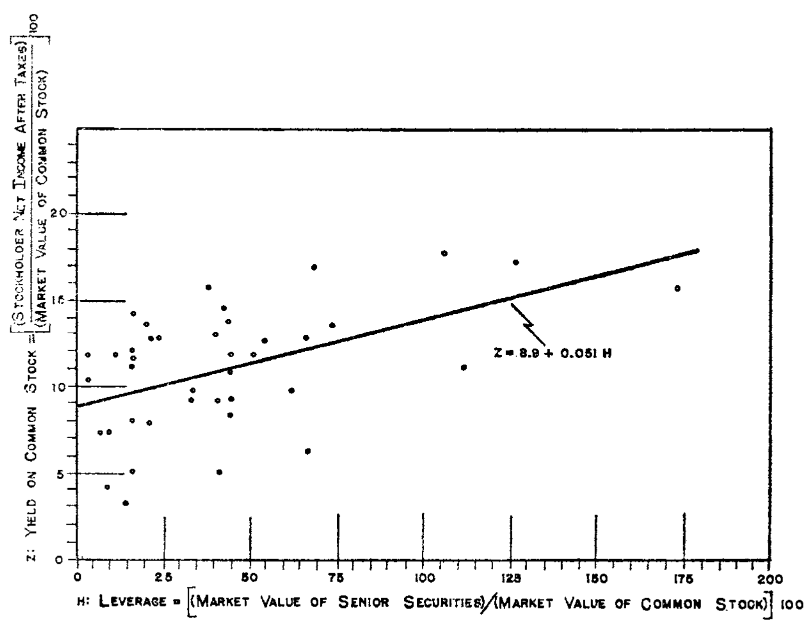

FIGURE 6. YIELD ON COMMON STOCK IN RELATION TO LEVERAGE FOR 42 OIL COMPANIES, 1952–53

the slope should be just above 2 per cent. The actual regression estimate for the slope of 1.7 per cent is thus somewhat low, but still within one standard error of its theoretical value. Because of this underestimate of the slope and because of the large mean value of leverage ($\bar{h}=160$ per cent) the regression estimate of the constant term, 6.6 per cent, is somewhat high, although not significantly different from the value of 5.6 per cent obtained in the test of Proposition I.

When we add a square term to the above equations to test for the presence and direction of curvature we obtain the following estimates:

$$\text{Electric Utilities} \quad z = 4.6 + .004h - .007h^2$$

$$\text{Oil Companies} \quad z = 8.5 + .072h - .016h^2.$$

For both cases the curvature is negative. In fact, for the electric utilities, where the observations cover a wider range of leverage ratios, the negative coefficient of the square term is actually significant at the 5 per cent level. Negative curvature, as we have seen, runs directly counter to the traditional hypothesis, whereas it can be readily accounted for by our model in terms of rising cost of borrowed funds.[41]

In summary, the empirical evidence we have reviewed seems to be broadly consistent with our model and largely inconsistent with traditional views. Needless to say much more extensive testing will be required before we can firmly conclude that our theory describes market behavior. Caution is indicated especially with regard to our test of Proposition II, partly because of possible statistical pitfalls[42] and partly because not all the factors that might have a systematic effect on stock yields have been considered. In particular, no attempt was made to test the possible influence of the dividend pay-out ratio whose role has tended to receive a great deal of attention in current research and thinking. There are two reasons for this omission. First, our main objective has been to assess the prima facie tenability of *our* model, and in this model, based as it is on rational behavior by investors, dividends per se play no role. Second, in a world in which the policy of dividend stabilization is widespread, there is no simple way of disentangling the true effect of dividend payments on stock prices from their apparent effect,

[41] That the yield of senior capital tended to rise for utilities as leverage increased is clearly shown in several of the scatter diagrams presented in the published version of Allen's study. This significant negative curvature between stock yields and leverage for utilities may be partly responsible for the fact, previously noted, that the constant in the linear regression is somewhat higher and the slope somewhat lower than implied by equation (12). Note also in connection with the estimate of ρ_k^7 that the introduction of the quadratic term reduces the constant considerably, pushing it in fact below the a priori expectation of 5.6, though the difference is again not statistically significant.

[42] In our test, *e.g.*, the two variables z and h are both ratios with S appearing in the denominator, which may tend to impart a positive bias to the correlation (*cf.* note 38). Attempts were made to develop alternative tests, but although various possibilities were explored, we have so far been unable to find satisfactory alternatives.

the latter reflecting only the role of dividends as a proxy measure of long-term earning anticipations.[43] The difficulties just mentioned are further compounded by possible interrelations between dividend policy and leverage.[44]

II. *Implications of the Analysis for the Theory of Investment*

A. *Capital Structure and Investment Policy*

On the basis of our propositions with respect to cost of capital and financial structure (and for the moment neglecting taxes), we can derive the following simple rule for optimal investment policy by the firm:

Proposition III. If a firm in class k is acting in the best interest of the stockholders at the time of the decision, it will exploit an investment opportunity if and only if the rate of return on the investment, say ρ^*, is as large as or larger than ρ_k. That is, ***the cut-off point for investment in the firm will in all cases be ρ_k and will be completely unaffected by the type of security used to finance the investment.*** Equivalently, we may say that regardless of the financing used, the marginal cost of capital to a firm is equal to the average cost of capital, which is in turn equal to the capitalization rate for an unlevered stream in the class to which the firm belongs.[45]

To establish this result we will consider the three major financing alternatives open to the firm—bonds, retained earnings, and common stock issues—and show that in each case an investment is worth undertaking if, and only if, $\rho^* \geqq \rho_k$.[46]

Consider first the case of an investment financed by the sale of bonds. We know from Proposition I that the market value of the firm before the investment was undertaken was:[47]

$$V_0 = \overline{X}_0/\rho_k \tag{20}$$

[43] We suggest that failure to appreciate this difficulty is responsible for many fallacious, or at least unwarranted, conclusions about the role of dividends.

[44] In the sample of electric utilities, there is a substantial negative correlation between yields and pay-out ratios, but also between pay-out ratios and leverage, suggesting that either the association of yields and leverage or of yields and pay-out ratios may be (at least partly) spurious. These difficulties however do not arise in the case of the oil industry sample. A preliminary analysis indicates that there is here no significant relation between leverage and pay-out ratios and also no significant correlation (either gross or partial) between yields and pay-out ratios.

[45] The analysis developed in this paper is essentially a comparative-statics, not a dynamic analysis. This note of caution applies with special force to Proposition III. Such problems as those posed by expected changes in r and in ρ_k over time will not be treated here. Although they are in principle amenable to analysis within the general framework we have laid out, such an undertaking is sufficiently complex to deserve separate treatment. *Cf.* note 17.

[46] The extension of the proof to other types of financing, such as the sale of preferred stock or the issuance of stock rights is straightforward.

[47] Since no confusion is likely to arise, we have again, for simplicity, eliminated the subscripts identifying the firm in the equations to follow. Except for ρ_k, the subscripts now refer to time periods.

and that the value of the common stock was:

$$S_0 = V_0 - D_0. \tag{21}$$

If now the firm borrows I dollars to finance an investment yielding ρ^* its market value will become:

$$V_1 = \frac{\overline{X}_0 + \rho^* I}{\rho_k} = V_0 + \frac{\rho^* I}{\rho_k} \tag{22}$$

and the value of its common stock will be:

$$S_1 = V_1 - (D_0 + I) = V_0 + \frac{\rho^* I}{\rho_k} - D_0 - I \tag{23}$$

or using equation 21,

$$S_1 = S_0 + \frac{\rho^* I}{\rho_k} - I. \tag{24}$$

Hence $S_1 \gtreqless S_0$ as $\rho^* \gtreqless \rho_k$.[48]

To illustrate, suppose the capitalization rate for uncertain streams in the kth class is 10 per cent and the rate of interest is 4 per cent. Then if a given company had an expected income of 1,000 and if it were financed entirely by common stock we know from Proposition I that the market value of its stock would be 10,000. Assume now that the managers of the firm discover an investment opportunity which will require an outlay of 100 and which is expected to yield 8 per cent. At first sight this might appear to be a profitable opportunity since the expected return is double the interest cost. If, however, the management borrows the necessary 100 at 4 per cent, the total expected income of the company rises to 1,008 and the market value of the firm to 10,080. But the firm now will have 100 of bonds in its capital structure so that, paradoxically, the market value of the stock must actually be reduced from 10,000 to 9,980 as a consequence of this apparently profitable investment. Or, to put it another way, the gains from being able to tap cheap, borrowed funds are more than offset for the stockholders by the market's discounting of the stock for the added leverage assumed.

Consider next the case of retained earnings. Suppose that in the course of its operations the firm acquired I dollars of cash (without impairing

[48] In the case of bond-financing the rate of interest on bonds does not enter explicitly into the decision (assuming the firm borrows at the market rate of interest). This is true, moreover, given the conditions outlined in Section I.C, even though interest rates may be an increasing function of debt outstanding. To the extent that the firm borrowed at a rate other than the market rate the two I's in equation (24) would no longer be identical and an additional gain or loss, as the case might be, would accrue to the shareholders. It might also be noted in passing that permitting the two I's in (24) to take on different values provides a simple method for introducing underwriting expenses into the analysis.

the earning power of its assets). If the cash is distributed as a dividend to the stockholders their wealth W_0, after the distribution will be:

$$W_0 = S_0 + I = \frac{\overline{X}_0}{\rho_k} - D_0 + I \tag{25}$$

where $\overline{X}_0$ represents the expected return from the assets exclusive of the amount I in question. If however the funds are retained by the company and used to finance new assets whose expected rate of return is ρ^*, then the stockholders' wealth would become:

$$W_1 = S_1 = \frac{\overline{X}_0 + \rho^* I}{\rho_k} - D_0 = S_0 + \frac{\rho^* I}{\rho_k} \cdot \tag{26}$$

Clearly $W_1 \gtreqless W_0$ as $\rho^* \gtreqless \rho_k$ so that an investment financed by retained earnings raises the net worth of the owners if and only if $\rho^* > \rho_k$.[49]

Consider finally, the case of common-stock financing. Let P_0 denote the current market price per share of stock and assume, for simplicity, that this price reflects currently expected earnings only, that is, it does not reflect any future increase in earnings as a result of the investment under consideration.[50] Then if N is the original number of shares, the price per share is:

$$P_0 = S_0/N \tag{27}$$

and the number of new shares, M, needed to finance an investment of I dollars is given by:

$$M = \frac{I}{P_0} \cdot \tag{28}$$

As a result of the investment the market value of the stock becomes:

$$S_1 = \frac{\overline{X}_0 + \rho^* I}{\rho_k} - D_0 = S_0 + \frac{\rho^* I}{\rho_k} = NP_0 + \frac{\rho^* I}{\rho_k}$$

and the price per share:

$$P_1 = \frac{S_1}{N + M} = \frac{1}{N + M}\left[NP_0 + \frac{\rho^* I}{\rho_k}\right]. \tag{29}$$

[49] The conclusion that ρ_k is the cut-off point for investments financed from internal funds applies not only to undistributed net profits, but to depreciation allowances (and even to the funds represented by the current sale value of any asset or collection of assets). Since the owners can earn ρ_k by investing funds elsewhere in the class, partial or total liquidating distributions should be made whenever the firm cannot achieve a marginal internal rate of return equal to ρ_k.

[50] If we assumed that the market price of the stock did reflect the expected higher future earnings (as would be the case if our original set of assumptions above were strictly followed) the analysis would differ slightly in detail, but not in essentials. The cut-off point for new investment would still be ρ_k, but where $\rho^* > \rho_k$ the gain to the original owners would be larger than if the stock price were based on the pre-investment expectations only.

Since by equation (28), $I = MP_0$, we can add MP_0 and subtract I from the quantity in bracket, obtaining:

$$
\begin{aligned}
P_1 &= \frac{1}{N+M}\left[(N+M)P_0 + \frac{\rho^* - \rho_k}{\rho_k} I\right] \\
&= P_0 + \frac{1}{N+M}\,\frac{\rho^* - \rho_k}{\rho_k} I > P_0 \text{ if,}
\end{aligned} \tag{30}
$$

and only if, $\rho^* > \rho_k$.

Thus an investment financed by common stock is advantageous to the current stockholders if and only if its yield exceeds the capitalization rate ρ_k.

Once again a numerical example may help to illustrate the result and make it clear why the relevant cut-off rate is ρ_k and not the current yield on common stock, i. Suppose that ρ_k is 10 per cent, r is 4 per cent, that the original expected income of our company is 1,000 and that management has the opportunity of investing 100 having an expected yield of 12 per cent. If the original capital structure is 50 per cent debt and 50 per cent equity, and 1,000 shares of stock are initially outstanding, then, by Proposition I, the market value of the common stock must be 5,000 or 5 per share. Furthermore, since the interest bill is $.04 \times 5{,}000 = 200$, the yield on common stock is $800/5{,}000 = 16$ per cent. It may then appear that financing the additional investment of 100 by issuing 20 shares to outsiders at 5 per share would dilute the equity of the original owners since the 100 promises to yield 12 per cent whereas the common stock is currently yielding 16 per cent. Actually, however, the income of the company would rise to 1,012; the value of the firm to 10,120; and the value of the common stock to 5,120. Since there are now 1,020 shares, each would be worth 5.02 and the wealth of the original stockholders would thus have been increased. What has happened is that the dilution in expected earnings per share (from .80 to .796) has been more than offset, in its effect upon the market price of the shares, by the decrease in leverage.

Our conclusion is, once again, at variance with conventional views,[51] so much so as to be easily misinterpreted. Read hastily, Proposition III seems to imply that the capital structure of a firm is a matter of indifference; and that, consequently, one of the core problems of corporate finance—the problem of the optimal capital structure for a firm—is no problem at all. It may be helpful, therefore, to clear up such possible misundertandings.

[51] In the matter of investment policy under uncertainty there is no single position which represents "accepted" doctrine. For a sample of current formulations, all very different from ours, see Joel Dean [2, esp. Ch. 3], M. Gordon and E. Shapiro [5], and Harry Roberts [17].

B. *Proposition III and Financial Planning by Firms*

Misinterpretation of the scope of Proposition III can be avoided by remembering that this Proposition tells us only that the type of instrument used to finance an investment is irrelevant to the question of whether or not the investment is worth while. This does not mean that the owners (or the managers) have no grounds whatever for preferring one financing plan to another; or that there are no other policy or technical issues in finance at the level of the firm.

That grounds for preferring one type of financial structure to another will still exist within the framework of our model can readily be seen for the case of common-stock financing. In general, except for something like a widely publicized oil-strike, we would expect the market to place very heavy weight on current and recent past earnings in forming expectations as to future returns. Hence, if the owners of a firm discovered a major investment opportunity which they felt would yield much more than ρ_k, they might well prefer not to finance it via common stock at the then ruling price, because this price may fail to capitalize the new venture. A better course would be a pre-emptive issue of stock (and in this connection it should be remembered that stockholders are free to borrow and buy). Another possibility would be to finance the project initially with debt. Once the project had reflected itself in increased actual earnings, the debt could be retired either with an equity issue at much better prices or through retained earnings. Still another possibility along the same lines might be to combine the two steps by means of a convertible debenture or preferred stock, perhaps with a progressively declining conversion rate. Even such a double-stage financing plan may possibly be regarded as yielding too large a share to outsiders since the new stockholders are, in effect, being given an interest in any similar opportunities the firm may discover in the future. If there is a reasonable prospect that even larger opportunities may arise in the near future and if there is some danger that borrowing now would preclude more borrowing later, the owners might find their interests best protected by splitting off the current opportunity into a separate subsidiary with independent financing. Clearly the problems involved in making the crucial estimates and in planning the optimal financial strategy are by no means trivial, even though they should have no bearing on the basic decision to invest (as long as $\rho^* \geqq \rho_k$).[52]

Another reason why the alternatives in financial plans may not be a matter of indifference arises from the fact that managers are concerned

[52] Nor can we rule out the possibility that the existing owners, if unable to use a financing plan which protects their interest, may actually prefer to pass up an otherwise profitable venture rather than give outsiders an "excessive" share of the business. It is presumably in situations of this kind that we could justifiably speak of a shortage of "equity capital," though this kind of market imperfection is likely to be of significance only for small or new firms.

with more than simply furthering the interest of the owners. Such other objectives of the management—which need not be necessarily in conflict with those of the owners—are much more likely to be served by some types of financing arrangements than others. In many forms of borrowing agreements, for example, creditors are able to stipulate terms which the current management may regard as infringing on its prerogatives or restricting its freedom to maneuver. The creditors might even be able to insist on having a direct voice in the formation of policy.[53] To the extent, therefore, that financial policies have these implications for the management of the firm, something like the utility approach described in the introductory section becomes relevant to financial (as opposed to investment) decision-making. It is, however, the utility functions of the managers per se and not of the owners that are now involved.[54]

In summary, many of the specific considerations which bulk so large in traditional discussions of corporate finance can readily be superimposed on our simple framework without forcing any drastic (and certainly no systematic) alteration of the conclusion which is our principal concern, namely that for investment decisions, the marginal cost of capital is ρ_k.

C. *The Effect of the Corporate Income Tax on Investment Decisions*

In Section I it was shown that when an unintegrated corporate income tax is introduced, the original version of our Proposition I,

$$\bar{X}/V = \rho_k = \text{a constant}$$

must be rewritten as:

$$\frac{(\bar{X} - rD)(1 - \tau) + rD}{V} \equiv \frac{\bar{X}^\tau}{V} = \rho_k{}^\tau = \text{a constant.} \tag{11}$$

Throughout Section I we found it convenient to refer to $\bar{X}^\tau/V$ as the cost of capital. The appropriate measure of the cost of capital relevant

[53] Similar considerations are involved in the matter of dividend policy. Even though the stockholders may be indifferent as to payout policy as long as investment policy is optimal, the management need not be so. Retained earnings involve far fewer threats to control than any of the alternative sources of funds and, of course, involve no underwriting expense or risk. But against these advantages management must balance the fact that sharp changes in dividend rates, which heavy reliance on retained earnings might imply, may give the impression that a firm's finances are being poorly managed, with consequent threats to the control and professional standing of the management.

[54] In principle, at least, this introduction of management's risk preferences with respect to financing methods would do much to reconcile the apparent conflict between Proposition III and such empirical findings as those of Modigliani and Zeman [14] on the close relation between interest rates and the ratio of new debt to new equity issues; or of John Lintner [12] on the considerable stability in target and actual dividend-payout ratios.

to investment decisions, however, is the ratio of the expected return *before* taxes to the market value, *i.e.*, $\overline{X}/V$. From (11) above we find:

$$\frac{\overline{X}}{V} = \frac{\rho_k{}^\tau - \tau_r(D/V)}{1-\tau} = \frac{\rho_k{}^\tau}{1-\tau}\left[1 - \frac{\tau r D}{\rho_k{}^\tau V}\right], \tag{31}$$

which shows that the cost of capital now depends on the debt ratio, decreasing, as D/V rises, at the constant rate $\tau r/(1-\tau)$.[55] Thus, with a corporate income tax under which interest is a deductible expense, gains can accrue to stockholders from having debt in the capital structure, even when capital markets are perfect. The gains however are small, as can be seen from (31), and as will be shown more explicitly below.

From (31) we can develop the tax-adjusted counterpart of Proposition III by interpreting the term D/V in that equation as the proportion of debt used in any additional financing of V dollars. For example, in the case where the financing is entirely by new common stock, $D=0$ and the required rate of return $\rho_k{}^S$ on a venture so financed becomes:

$$\rho_k{}^S = \frac{\rho_k{}^\tau}{1-\tau}. \tag{32}$$

For the other extreme of pure debt financing $D=V$ and the required rate of return, $\rho_k{}^D$, becomes:

$$\rho_k{}^D = \frac{\rho_k{}^\tau}{1-\tau}\left[1 - \tau\frac{r}{\rho_k{}^\tau}\right] = \rho_k{}^S\left[1 - \tau\frac{r}{\rho_k{}^\tau}\right] = \rho_k{}^S - \frac{\tau}{1-\tau}r.\text{[56]} \tag{33}$$

For investments financed out of retained earnings, the problem of defining the required rate of return is more difficult since it involves a comparison of the tax consequences to the individual stockholder of receiving a dividend versus having a capital gain. Depending on the time of realization, a capital gain produced by retained earnings may be taxed either at ordinary income tax rates, 50 per cent of these rates, 25 per

[55] Equation (31) is amenable, in principle, to statistical tests similar to those described in Section I.E. However we have not made any systematic attempt to carry out such tests so far, because neither the Allen nor the Smith study provides the required information. Actually, Smith's data included a very crude estimate of tax liability, and, using this estimate, we did in fact obtain a negative relation between $\overline{X}/V$ and D/V. However, the correlation ($-.28$) turned out to be significant only at about the 10 per cent level. While this result is not conclusive, it should be remembered that, according to our theory, the slope of the regression equation should be in any event quite small. In fact, with a value of τ in the order of .5, and values of $\rho_k{}^\tau$ and r in the order of 8.5 and 3.5 per cent respectively (*cf.* Section I.E) an increase in D/V from 0 to 60 per cent (which is, approximately, the range of variation of this variable in the sample) should tend to reduce the average cost of capital only from about 17 to about 15 per cent.

[56] This conclusion does not extend to preferred stocks even though they have been classed with debt issues previously. Since preferred dividends except for a portion of those of public utilities are not in general deductible from the corporate tax, the cut-off point for new financing via preferred stock is exactly the same as that for common stock.

cent, or zero, if held till death. The rate on any dividends received in the event of a distribution will also be a variable depending on the amount of other income received by the stockholder, and with the added complications introduced by the current dividend-credit provisions. If we assume that the managers proceed on the basis of reasonable estimates as to the average values of the relevant tax rates for the owners, then the required return for retained earnings $\rho_k{}^R$ can be shown to be:

$$\rho_k{}^R = \rho_k{}^\tau \frac{1}{1-\tau} \frac{1-\tau_d}{1-\tau_g} = \frac{1-\tau_d}{1-\tau_g} \rho_k{}^* \tag{34}$$

where τ_d is the assumed rate of personal income tax on dividends and τ_g is the assumed rate of tax on capital gains.

A numerical illustration may perhaps be helpful in clarifying the relationship between these required rates of return. If we take the following round numbers as representative order-of-magnitude values under present conditions: an after-tax capitalization rate $\rho_k{}^\tau$ of 10 per cent, a rate of interest on bonds of 4 per cent, a corporate tax rate of 50 per cent, a marginal personal income tax rate on dividends of 40 per cent (corresponding to an income of about $25,000 on a joint return), and a capital gains rate of 20 per cent (one-half the marginal rate on dividends), then the required rates of return would be: (1) 20 per cent for investments financed entirely by issuance of new common shares; (2) 16 per cent for investments financed entirely by new debt; and (3) 15 per cent for investments financed wholly from internal funds.

These results would seem to have considerable significance for current discussions of the effect of the corporate income tax on financial policy and on investment. Although we cannot explore the implications of the results in any detail here, we should at least like to call attention to the remarkably small difference between the "cost" of equity funds and debt funds. With the numerical values assumed, equity money turned out to be only 25 per cent more expensive than debt money, rather than something on the order of 5 times as expensive as is commonly supposed to be the case.[57] The reason for the wide difference is that the traditional

[57] See *e.g.*, D. T. Smith [18]. It should also be pointed out that our tax system acts in other ways to reduce the gains from debt financing. Heavy reliance on debt in the capital structure, for example, commits a company to paying out a substantial proportion of its income in the form of interest payments taxable to the owners under the personal income tax. A debt-free company, by contrast, can reinvest in the business all of its (smaller) net income and to this extent subject the owners only to the low capital gains rate (or possibly no tax at all by virtue of the loophole at death). Thus, we should expect a high degree of leverage to be of value to the owners, even in the case of closely held corporations, primarily in cases where their firm was not expected to have much need for additional funds to expand assets and earnings in the future. To the extent that opportunities for growth were available, as they presumably would be for most successful corporations, the interest of the stockholders would tend to be better served by a structure which permitted maximum use of retained earnings.

view starts from the position that debt funds are several times cheaper than equity funds even in the absence of taxes, with taxes serving simply to magnify the cost ratio in proportion to the corporate rate. By contrast, in our model in which the repercussions of debt financing on the value of shares are taken into account, the *only* difference in cost is that due to the tax effect, and its magnitude is simply the tax on the "grossed up" interest payment. Not only is this magnitude likely to be small but our analysis yields the further paradoxical implication that the stockholders' gain from, and hence incentive to use, debt financing is actually smaller the lower the rate of interest. In the extreme case where the firm could borrow for practically nothing, the advantage of debt financing would also be practically nothing.

III. *Conclusion*

With the development of Proposition III the main objectives we outlined in our introductory discussion have been reached. We have in our Propositions I and II at least the foundations of a theory of the valuation of firms and shares in a world of uncertainty. We have shown, moreover, how this theory can lead to an operational definition of the cost of capital and how that concept can be used in turn as a basis for rational investment decision-making within the firm. Needless to say, however, much remains to be done before the cost of capital can be put away on the shelf among the solved problems. Our approach has been that of static, partial equilibrium analysis. It has assumed among other things a state of atomistic competition in the capital markets and an ease of access to those markets which only a relatively small (though important) group of firms even come close to possessing. These and other drastic simplifications have been necessary in order to come to grips with the problem at all. Having served their purpose they can now be relaxed in the direction of greater realism and relevance, a task in which we hope others interested in this area will wish to share.

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Corporate Income Taxes and the Cost of Capital: A Correction

Franco Modigliani; Merton H. Miller

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equanimity a writing-down of the value of their reserves, or unless one is prepared to forego the possibility of exchange-rate adjustment, any major extension of the gold exchange standard is dependent upon the introduction of guarantees. It is misleading to suggest that the multiple key-currency system is an alternative to a guarantee, as implied by Roosa [6, pp. 5-7 and 9-12].

IV. *Conclusion*

The most noteworthy conclusion to be drawn from this analysis is that the successful operation of a multiple key-currency system would require both exchange guarantees and continuing cooperation between central bankers of a type that would effectively limit their choice as to the form in which they hold their reserves. Yet these are two of the conditions whose undesirability has frequently been held to be an obstacle to implementation of the alternative proposal to create a world central bank. The multiple key-currency proposal represents an attempt to avoid the impracticality supposedly associated with a world central bank, but if both proposals in fact depend on the fulfillment of similar conditions, it is difficult to convince oneself that the sacrifice of the additional liquidity that an almost closed system would permit is worth while. Unless, of course, the object of the exercise is to reinforce discipline rather than to expand liquidity.

JOHN WILLIAMSON*

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* The author is instructor in economics at Princeton University. He acknowledges the helpful comments of Fritz Machlup. Views expressed are those of the author alone.

Corporate Income Taxes and the Cost of Capital: A Correction

The purpose of this communication is to correct an error in our paper "The Cost of Capital, Corporation Finance and the Theory of Investment" (this *Review*, June 1958). In our discussion of the effects of the present method of taxing corporations on the valuation of firms, we said (p. 272):

> The deduction of interest in computing taxable corporate profits will prevent the arbitrage process from making the value of all firms in a given class proportional to the expected returns generated by their

physical assets. Instead, it can be shown (by the same type of proof used for the original version of Proposition I) that *the market values of firms in each class must be proportional in equilibrium to their expected returns net of taxes* (*that is, to the sum of the interest paid and expected net stockholder income*). (Italics added.)

The statement in italics, unfortunately, is wrong. For even though one firm may have an *expected* return after taxes (our $\bar{X}^\tau$) twice that of another firm in the same risk-equivalent class, it will not be the case that the *actual* return after taxes (our X^τ) of the first firm will always be twice that of the second, if the two firms have different degrees of leverage.[1] And since the distribution of returns after taxes of the two firms will not be proportional, there can be no "arbitrage" process which forces their values to be proportional to their expected after-tax returns.[2] In fact, it can be shown—and this time it really will be shown—that "arbitrage" will make values within any class a function not only of expected after-tax returns, but of the tax rate and the degree of leverage. This means, among other things, that the tax advantages of debt financing are somewhat greater than we originally suggested and, to this extent, the quantitative difference between the valuations implied by our position and by the traditional view is narrowed. It still remains true, however, that under our analysis the tax advantages of debt are the *only* permanent advantages so that the gulf between the two views in matters of interpretation and policy is as wide as ever.

I. *Taxes, Leverage, and the Probability Distribution of After-Tax Returns*

To see how the distribution of after-tax earnings is affected by leverage, let us again denote by the random variable X the (long-run average) earnings before interest and taxes generated by the currently owned assets of a given firm in some stated risk class, k.[3] From our definition of a risk class it follows that X can be expressed in the form $\bar{X}Z$, where $\bar{X}$ is the expected value of X, and the random variable $Z = X/\bar{X}$, having the same value for all firms in class k, is a drawing from a distribution, say $f_k(Z)$. Hence the

[1] With some exceptions, which will be noted when they occur, we shall preserve here both the notation and the terminology of the original paper. A working knowledge of both on the part of the reader will be presumed.

[2] Barring, of course, the trivial case of universal linear utility functions. Note that in deference to Professor Durand (see his Comment on our paper and our reply, this *Review*, Sept. 1959, *49*, 639–69) we here and throughout use quotation marks when referring to arbitrage.

[3] Thus our X corresponds essentially to the familiar EBIT concept of the finance literature. The use of EBIT and related "income" concepts as the basis of valuation is strictly valid only when the underlying real assets are assumed to have perpetual lives. In such a case, of course, EBIT and "cash flow" are one and the same. This was, in effect, the interpretation of X we used in the original paper and we shall retain it here both to preserve continuity and for the considerable simplification it permits in the exposition. We should point out, however, that the perpetuity interpretation is much less restrictive than might appear at first glance. Before-tax cash flow and EBIT can also safely be equated even where assets have finite lives as soon as these assets attain a steady state age distribution in which annual replacements equal annual depreciation. The subject of finite lives of assets will be further discussed in connection with the problem of the cut-off rate for investment decisions.

random variable X^τ, measuring the after-tax return, can be expressed as:

$$(1)\quad X^\tau = (1-\tau)(X-R)+R = (1-\tau)X+\tau R = (1-\tau)\overline{X}Z+\tau R$$

where τ is the marginal corporate income tax rate (assumed equal to the average), and R is the interest bill. Since $E(X^\tau) \equiv \overline{X}^\tau = (1-\tau)\overline{X}+\tau R$ we can substitute $\overline{X}^\tau - \tau R$ for $(1-\tau)\overline{X}$ in (1) to obtain:

$$(2)\qquad X^\tau = (\overline{X}^\tau - \tau R)Z + \tau R = \overline{X}^\tau\left(1-\frac{\tau R}{\overline{X}^\tau}\right)Z+\tau R.$$

Thus, if the tax rate is other than zero, the shape of the distribution of X^τ will depend not only on the "scale" of the stream $\overline{X}^\tau$ and on the distribution of Z, but also on the tax rate and the degree of leverage (one measure of which is $R/\overline{X}^\tau$). For example, if Var $(Z)=\sigma^2$, we have:

$$\text{Var}\,(X^\tau) = \sigma^2(\overline{X}^\tau)^2\left(1-\tau\frac{R}{\overline{X}^\tau}\right)^2$$

implying that for given $\overline{X}^\tau$ the variance of after-tax returns is smaller, the higher τ and the degree of leverage.[4]

II. *The Valuation of After-Tax Returns*

Note from equation (1) that, from the investor's point of view, the long-run average stream of after-tax returns appears as a sum of two components: (1) an uncertain stream $(1-\tau)\overline{X}Z$; and (2) a sure stream τR.[5] This suggests that the equilibrium market value of the combined stream can be found by capitalizing each component separately. More precisely, let ρ^τ be the rate at which the market capitalizes the expected returns net of tax of an unlevered company of size $\overline{X}$ in class k, i.e.,

$$\rho^\tau = \frac{(1-\tau)\overline{X}}{V_U} \quad \text{or} \quad V_U = \frac{(1-\tau)\overline{X}}{\rho^\tau};^{6}$$

[4] It may seem paradoxical at first to say that leverage *reduces* the variability of outcomes, but remember we are here discussing the variability of total returns, interest plus net profits. The variability of stockholder net profits will, of course, be greater in the presence than in the absence of leverage, though relatively less so than in an otherwise comparable world of no taxes. The reasons for this will become clearer after the discussion in the next section.

[5] The statement that τR—the tax saving per period on the interest payments—is a sure stream is subject to two qualifications. First, it must be the case that firms can always obtain the tax benefit of their interest deductions either by offsetting them directly against other taxable income in the year incurred; or, in the event no such income is available in any given year, by carrying them backward or forward against past or future taxable earnings; or, in the extreme case, by merger of the firm with (or its sale to) another firm that can utilize the deduction. Second, it must be assumed that the tax rate will remain the same. To the extent that neither of these conditions holds exactly then some uncertainty attaches even to the tax savings, though, of course, it is of a different kind and order from that attaching to the stream generated by the assets. For simplicity, however, we shall here ignore these possible elements of delay or of uncertainty in the tax saving; but it should be kept in mind that this neglect means that the subsequent valuation formulas overstate, if anything, the value of the tax saving for any given permanent level of debt.

[6] Note that here, as in our original paper, we neglect dividend policy and "growth" in the

and let r be the rate at which the market capitalizes the sure streams generated by debts. For simplicity, assume this rate of interest is a constant independent of the size of the debt so that

$$r = \frac{R}{D} \quad \text{or} \quad D = \frac{R}{r}.$$[7]

Then we would expect the value of a levered firm of size $\overline{X}$, with a permanent level of debt D_L in its capital structure, to be given by:

$$V_L = \frac{(1-\tau)\overline{X}}{\rho^\tau} + \frac{\tau R}{r} = V_U + \tau D_L. \tag{3}$$[8]

In our original paper we asserted instead that, within a risk class, market value would be proportional to expected after-tax return $\overline{X}^\tau$ (cf. our original equation [11]), which would imply:

$$V_L = \frac{\overline{X}^\tau}{\rho^\tau} = \frac{(1-\tau)\overline{X}}{\rho^\tau} + \frac{\tau R}{\rho^\tau} = V_U + \frac{r}{\rho^\tau}\tau D_L. \tag{4}$$

We will now show that if (3) does not hold, investors can secure a more efficient portfolio by switching from relatively overvalued to relatively undervalued firms. Suppose first that unlevered firms are overvalued or that

$$V_L - \tau D_L < V_U.$$

An investor holding m dollars of stock in the unlevered company has a right to the fraction m/V_U of the eventual outcome, i.e., has the uncertain income

$$Y_U = \left(\frac{m}{V_U}\right)(1-\tau)\overline{X}Z.$$

Consider now an alternative portfolio obtained by investing m dollars as follows: (1) the portion,

$$m\left(\frac{S_L}{S_L + (1-\tau)D_L}\right),$$

is invested in the stock of the levered firm, S_L; and (2) the remaining portion,

$$m\left(\frac{(1-\tau)D_L}{S_L + (1-\tau)D_L}\right),$$

sense of opportunities to invest at a rate of return greater than the market rate of return. These subjects are treated extensively in our paper, "Dividend Policy, Growth and the Valuation of Shares," *Jour. Bus.*, Univ. Chicago, Oct. 1961, 411–33.

[7] Here and throughout, the corresponding formulas when the rate of interest rises with leverage can be obtained merely by substituting $r(L)$ for r, where L is some suitable measure of leverage.

[8] The assumption that the debt is permanent is not necessary for the analysis. It is employed here both to maintain continuity with the original model and because it gives an upper bound on the value of the tax saving. See in this connection footnote 5 and footnote 9.

is invested in its bonds. The stock component entitles the holder to a fraction,

$$\frac{m}{S_L + (1 - \tau) D_L},$$

of the net profits of the levered company or

$$\left(\frac{m}{S_L + (1 - \tau) D_L}\right) [(1 - \tau)(\overline{X}Z - R_L)].$$

The holding of bonds yields

$$\left(\frac{m}{S_L + (1 - \tau) D_L}\right) [(1 - \tau) R_L].$$

Hence the total outcome is

$$Y_L = \left(\frac{m}{(S_L + (1 - \tau) D_L)}\right) [(1 - \tau)\overline{X}Z]$$

and this will dominate the uncertain income Y_U if (and only if)

$$S_L + (1 - \tau) D_L \equiv S_L + D_L - \tau D_L \equiv V_L - \tau D_L < V_U.$$

Thus, in equilibrium, V_U cannot exceed $V_L - \tau D_L$, for if it did investors would have an incentive to sell shares in the unlevered company and purchase the shares (and bonds) of the levered company.

Suppose now that $V_L - \tau D_L > V_U$. An investment of m dollars in the stock of the levered firm entitles the holder to the outcome

$$\begin{aligned} Y_L &= (m/S_L)[(1 - \tau)(\overline{X}Z - R_L)] \\ &= (m/S_L)(1 - \tau)\overline{X}Z - (m/S_L)(1 - \tau) R_L. \end{aligned}$$

Consider the following alternative portfolio: (1) borrow an amount $(m/S_L)(1 - \tau) D_L$ for which the interest cost will be $(m/S_L)(1 - \tau) R_L$ (assuming, of course, that individuals and corporations can borrow at the same rate, r); and (2) invest m plus the amount borrowed, i.e.,

$$m + \frac{m(1 - \tau) D_L}{S_L} = m \frac{S_L + (1 - \tau) D_L}{S_L} = (m/S_L)[V_L - \tau D_L]$$

in the stock of the unlevered firm. The outcome so secured will be

$$(m/S_L)\left(\frac{V_L - \tau D_L}{V_U}\right)(1 - \tau)\overline{X}Z.$$

Subtracting the interest charges on the borrowed funds leaves an income of

$$Y_U = (m/S_L)\left(\frac{V_L - \tau D_L}{V_U}\right)(1 - \tau)\overline{X}Z - (m/S_L)(1 - \tau) R_L$$

which will dominate Y_L if (and only if) $V_L - \tau D_L > V_U$. Thus, in equilibrium, both $V_L - \tau D_L > V_U$ and $V_L - \tau D_L < V_U$ are ruled out and (3) must hold.

III. *Some Implications of Formula* (3)

To see what is involved in replacing (4) with (3) as the rule of valuation, note first that both expressions make the value of the firm a function of leverage and the tax rate. The difference between them is a matter of the size and source of the tax advantages of debt financing. Under our original formulation, values within a class were strictly proportional to expected earnings after taxes. Hence the tax advantage of debt was due solely to the fact that the deductibility of interest payments implied a higher level of after-tax income for any given level of before-tax earnings (i.e., higher by the amount τR since $\bar{X}^{\tau} = (1-\tau)\bar{X} + \tau R$). Under the corrected rule (3), however, there is an additional gain due to the fact that the extra after-tax earnings, τR, represent a sure income in contrast to the uncertain outcome $(1-\tau)\bar{X}$. Hence τR is capitalized at the more favorable certainty rate, $1/r$, rather than at the rate for uncertain streams, $1/\rho^{\tau}$.[9]

Since the difference between (3) and (4) is solely a matter of the rate at which the tax savings on interest payments are capitalized, the required changes in all formulas and expressions derived from (4) are reasonably straightforward. Consider, first, the before-tax earnings yield, i.e., the ratio of expected earnings before interest and taxes to the value of the firm.[10] Dividing both sides of (3) by V and by $(1-\tau)$ and simplifying we obtain:

$$\text{(31.c)} \qquad \frac{\bar{X}}{V} = \frac{\rho^{\tau}}{1-\tau}\left[1 - \tau\frac{D}{V}\right]$$

which replaces our original equation (31) (p. 294). The new relation differs from the old in that the coefficient of D/V in the original (31) was smaller by a factor of r/ρ^{τ}.

Consider next the after-tax earnings yield, i.e., the ratio of interest payments plus profits after taxes to total market value.[11] This concept was discussed extensively in our paper because it helps to bring out more clearly the differences between our position and the traditional view, and because it facilitates the construction of empirical tests of the two hypotheses about the valuation process. To see what the new equation (3) implies for this yield we need merely substitute $\bar{X}^{\tau} - \tau R$ for $(1-\tau)\bar{X}$ in (3) obtaining:

[9] Remember, however, that in one sense formula (3) gives only an upper bound on the value of the firm since $\tau R/r = \tau D$ is an exact measure of the value of the tax saving only where both the tax rate and the level of debt are assumed to be fixed forever (and where the firm is certain to be able to use its interest deduction to reduce taxable income either directly or via transfer of the loss to another firm). Alternative versions of (3) can readily be developed for cases in which the debt is not assumed to be permanent, but rather to be outstanding only for some specified finite length of time. For reasons of space, we shall not pursue this line of inquiry here beyond observing that the shorter the debt period considered, the closer does the valuation formula approach our original (4). Hence, the latter is perhaps still of some interest if only as a lower bound.

[10] Following usage common in the field of finance we referred to this yield as the "average cost of capital." We feel now, however, that the term "before-tax earnings yield" would be preferable both because it is more immediately descriptive and because it releases the term "cost of capital" for use in discussions of optimal investment policy (in accord with standard usage in the capital budgeting literature).

[11] We referred to this yield as the "after-tax cost of capital." Cf. the previous footnote.

(5) $$V = \frac{\overline{X}^\tau - \tau R}{\rho^\tau} + \tau D = \frac{\overline{X}^\tau}{\rho^\tau} + \tau \frac{\rho^\tau - r}{\rho^\tau} D,$$

from which it follows that the after-tax earnings yield must be:

(11.c) $$\frac{\overline{X}^\tau}{V} = \rho^\tau - \tau(\rho^\tau - r)D/V.$$

This replaces our original equation (11) (p. 272) in which we had simply $\overline{X}^\tau/V = \rho^\tau$. Thus, in contrast to our earlier result, the corrected version (11.c) implies that even the after-tax yield is affected by leverage. The predicted rate of decrease of $\overline{X}^\tau/V$ with D/V, however, is still considerably smaller than under the naive traditional view, which, as we showed, implied essentially $\overline{X}^\tau/V = \rho^\tau\text{-}(\rho^\tau - r)D/V$. See our equation (17) and the discussion immediately preceding it (p. 277).[12] And, of course, (11.c) implies that the effect of leverage on $\overline{X}^\tau/V$ is *solely* a matter of the deductibility of interest payments whereas, under the traditional view, going into debt would lower the cost of capital regardless of the method of taxing corporate earnings.

Finally, we have the matter of the after-tax yield on *equity* capital, i.e., the ratio of net profits after taxes to the value of the shares.[13] By subtracting D from both sides of (5) and breaking $\overline{X}^\tau$ into its two components—expected net profits after taxes, $\bar{\pi}^\tau$, and interest payments, $R = rD$—we obtain after simplifying:

(6) $$S = V - D = \frac{\bar{\pi}^\tau}{\rho^\tau} - (1 - \tau)\left(\frac{\rho^\tau - r}{\rho^\tau}\right)D.$$

From (6) it follows that the after-tax yield on equity capital must be:

(12.c) $$\frac{\bar{\pi}^\tau}{S} = \rho^\tau + (1 - \tau)[\rho^\tau - r]D/S$$

which replaces our original equation (12), $\bar{\pi}^\tau/S = \rho^\tau + (\rho^\tau - r)D/S$ (p. 272). The new (12.c) implies an increase in the after-tax yield on equity capital as leverage increases which is smaller than that of our original (12) by a factor of $(1-\tau)$. But again, the linear increasing relation of the corrected (12.c) is still fundamentally different from the naive traditional view which asserts the cost of equity capital to be completely independent of leverage (at least as long as leverage remains within "conventional" industry limits).

IV. *Taxes and the Cost of Capital*

From these corrected valuation formulas we can readily derive corrected measures of the cost of capital in the capital budgeting sense of the minimum prospective yield an investment project must offer to be just worth

[12] The i_k^* of (17) is the same as ρ^τ in the present context, each measuring the ratio of net profits to the value of the shares (and hence of the whole firm) in an unlevered company of the class.

[13] We referred to this yield as the "after-tax cost of equity capital." Cf. footnote 9.

undertaking from the standpoint of the present stockholders. If we interpret earnings streams as perpetuities, as we did in the original paper, then we actually have two equally good ways of defining this minimum yield: either by the required increase in before-tax earnings, $d\bar{X}$, or by the required increase in earnings net of taxes, $d\bar{X}(1-\tau)$.[14] To conserve space, however, as well as to maintain continuity with the original paper, we shall concentrate here on the before-tax case with only brief footnote references to the net-of-tax concept.

Analytically, the derivation of the cost of capital in the above sense amounts to finding the minimum value of $d\bar{X}/dI$ for which $dV=dI$, where I denotes the level of new investment.[15] By differentiating (3) we see that:

$$\frac{dV}{dI} = \frac{1-\tau}{\rho^\tau}\frac{d\bar{X}}{dI} + \tau\frac{dD}{dI} \geq 1 \qquad \text{if } \frac{d\bar{X}}{dI} \geq \frac{1-\tau\dfrac{dD}{dI}}{1-\tau}\rho^\tau. \tag{7}$$

Hence the before tax required rate of return cannot be defined without reference to financial policy. In particular, for an investment considered as being financed entirely by new equity capital $dD/dI=0$ and the required rate of return or marginal cost of equity financing (neglecting flotation costs) would be:

$$\rho^S = \frac{\rho^\tau}{1-\tau}.$$

This result is the same as that in the original paper (see equation [32], p. 294) and is applicable to any other sources of financing where the remuneration to the suppliers of capital is not deductible for tax purposes. It applies, therefore, to preferred stock (except for certain partially deductible issues of public utilities) and would apply also to retained earnings were it not for the favorable tax treatment of capital gains under the personal income tax.

For investments considered as being financed entirely by new debt capital $dI=dD$ and we find from (7) that:

$$\rho^D = \rho^\tau \tag{33.c}$$

which replaces our original equation (33) in which we had:

$$\rho^D = \rho^S - \frac{\tau}{1-\tau}r. \tag{33}$$

[14] Note that we use the term "earnings net of taxes" rather than "earnings after taxes." We feel that to avoid confusion the latter term should be reserved to describe what will actually appear in the firm's accounting statements, namely the net cash flow including the tax savings on the interest (our $\bar{X}^\tau$). Since financing sources cannot in general be allocated to particular investments (see below), the after-tax or accounting concept is not useful for capital budgeting purposes, although it can be extremely useful for valuation equations as we saw in the previous section.

[15] Remember that when we speak of the minimum required yield on an investment we are referring in principle only to investments which increase the *scale* of the firm. That is, the new

Thus for borrowed funds (or any other tax-deductible source of capital) the marginal cost or before-tax required rate of return is simply the market rate of capitalization for net of tax unlevered streams and is thus independent of both the tax rate and the interest rate. This required rate is lower than that implied by our original (33), but still considerably higher than that implied by the traditional view (see esp. pp. 276–77 of our paper) under which the before-tax cost of borrowed funds is simply the interest rate, r.

Having derived the above expressions for the marginal costs of debt and equity financing it may be well to warn readers at this point that these expressions represent at best only the hypothetical extremes insofar as costs are concerned and that neither is directly usable as a cut-off criterion for investment planning. In particular, care must be taken to avoid falling into the famous "Liquigas" fallacy of concluding that if a firm intends to float a bond issue in some given year then its cut-off rate should be set that year at ρ^D; while, if the next issue is to be an equity one, the cut-off is ρ^S. The point is, of course, that no investment can meaningfully be regarded as 100 per cent equity financed if the firm makes any use of debt capital—and most firms do, not only for the tax savings, but for many other reasons having nothing to do with "cost" in the present static sense (cf. our original paper pp. 292–93). And no investment can meaningfully be regarded as 100 per cent debt financed when lenders impose strict limitations on the maximum amount a firm can borrow relative to its equity (and when most firms actually plan on normally borrowing less than this external maximum so as to leave themselves with an emergency reserve of unused borrowing power). Since the firm's long-run capital structure will thus contain both debt and equity capital, investment planning must recognize that, over the long pull, *all* of the firm's assets are really financed by a mixture of debt and equity capital even though only one kind of capital may be raised in any particular year. More precisely, if L^* denotes the firm's long-run "target" debt ratio (around which its actual debt ratio will fluctuate as it "alternately" floats debt issues and retires them with internal or external equity) then the firm can assume, to a first approximation at least, that for any particular investment $dD/dI = L^*$. Hence, the relevant marginal cost of capital for investment planning, which we shall here denote by ρ^*, is:

$$\rho^* = \frac{1 - \tau L^*}{1 - \tau} \rho^\tau = \rho^S - \frac{\tau}{1 - \tau} \rho^D L^* = \rho^S(1 - L^*) + \rho^D L^*.$$

That is, the appropriate cost of capital for (repetitive) investment decisions over time is, to a first approximation, a weighted average of the costs of debt and equity financing, the weights being the proportions of each in the "target" capital structure.[16]

assets must be in the same "class" as the old. See in this connection, J. Hirshleifer, "Risk, the Discount Rate and Investment Decisions," *Am. Econ. Rev.*, May 1961, *51*, 112–20 (especially pp. 119–20). See also footnote 16.

[16] From the formulas in the text one can readily derive corresponding expressions for the required net-of-tax yield, or net-of-tax cost of capital for any given financing policy. Specifi-

V. *Some Concluding Observations*

Such, then, are the major corrections that must be made to the various formulas and valuation expressions in our earlier paper. In general, we can say that the force of these corrections has been to increase somewhat the estimate of the tax advantages of debt financing under our model and consequently to reduce somewhat the quantitative difference between the estimates of the effects of leverage under our model and under the naive traditional view. It may be useful to remind readers once again that the existence of a tax advantage for debt financing—even the larger advantage of the corrected version—does not necessarily mean that corporations should at all times seek to use the maximum possible amount of debt in their capital structures. For one thing, other forms of financing, notably retained earnings, may in some circumstances be cheaper still when the tax status of investors under the personal income tax is taken into account. More important, there are, as we pointed out, limitations imposed by lenders (see pp. 292–93), as well as many other dimensions (and kinds of costs) in real-world problems of financial strategy which are not fully comprehended within the framework of static equilibrium models, either our own or those of the traditional variety. These additional considerations, which are typically grouped under the rubric of "the need for preserving flexibility," will normally imply the maintenance by the corporation of a substantial reserve of untapped borrowing power. The tax advantage of debt may well tend to lower the optimal size of that reserve, but it is hard to believe that advantages of the size contemplated under our model could justify any substantial reduction, let alone their complete elimination. Nor do the data

---

cally, let $\tilde{\rho}(L)$ denote the required net-of-tax yield for investment financed with a proportion of debt $L=dD/dI$. (More generally L denotes the proportion financed with tax deductible sources of capital.) Then from (7) we find:

$$\tilde{\rho}(L)=(1-\tau)\frac{d\bar{X}}{dI}=(1-L\tau)\rho^{\tau} \tag{8}$$

and the various costs can be found by substituting the appropriate value for L. In particular, if we substitute in this formula the "target" leverage ratio, L^*, we obtain:

$$\tilde{\rho}^*\equiv\tilde{\rho}(L^*)=(1-\tau L^*)\rho^{\tau}$$

and $\tilde{\rho}^*$ measures the average net-of-tax cost of capital in the sense described above.

Although the before-tax and the net-of-tax approaches to the cost of capital provide equally good criteria for investment decisions when assets are assumed to generate perpetual (i.e., non-depreciating) streams, such is not the case when assets are assumed to have finite lives (even when it is also assumed that the firm's assets are in a steady state age distribution so that our X or EBIT is approximately the same as the net cash flow before taxes). See footnote 3 above. In the latter event, the correct method for determining the desirability of an investment would be, in principle, to discount the net-of-tax stream at the net-of-tax cost of capital. Only under this net-of-tax approach would it be possible to take into account the deductibility of depreciation (and also to choose the most advantageous depreciation policy for tax purposes). Note that we say that the net-of-tax approach is correct "in principle" because, strictly speaking, nothing in our analysis (or anyone else's, for that matter) has yet established that it is indeed legitimate to "discount" an uncertain stream. One can hope that subsequent research will show the analogy to discounting under the certainty case is a valid one; but, at the moment, this is still only a hope.

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indicate that there has in fact been a substantial increase in the use of debt (except relative to preferred stock) by the corporate sector during the recent high tax years.[17]

As to the differences between our modified model and the traditional one, we feel that they are still large in quantitative terms and still very much worth trying to detect. It is not only a matter of the two views having different implications for corporate financial policy (or even for national tax policy). But since the two positions rest on fundamentally different views about investor behavior and the functioning of the capital markets, the results of tests between them may have an important bearing on issues ranging far beyond the immediate one of the effects of leverage on the cost of capital.

FRANCO MODIGLIANI AND MERTON H. MILLER*

[17] See, e.g., Merton H. Miller, "The Corporate Income Tax and Corporate Financial Policies," in *Staff Reports to the Commission on Money and Credit* (forthcoming).

* The authors are, respectively, professor of industrial management, School of Industrial Management, Massachusetts Institute of Technology, and professor of finance, Graduate School of Business, University of Chicago.

Consumption, Savings and Windfall Gains: Comment

In her recent article in this *Review* [3], Margaret Reid attempted to answer previous articles by Bodkin [1] and Jones [2] challenging the validity of the permanent income hypothesis. Bodkin and Jones used income and expenditure data for those consumer units who had received the soldiers' bonus (National Service Life Insurance dividends) during 1950, the year of the urban consumption survey [4]. These bonuses were regarded as windfall gains for the purposes of their analyses.

Professor Reid used data from the same survey, but her windfall gains were represented by "other money receipts." These are defined as "inheritances and occasional large gifts of money from persons outside the family . . . and net receipts from the settlement of fire and accident policies" [4, Vol. 1, p. xxix]. She assumed that the soldiers' bonus was included, and that it accounted for about one-half of other money receipts. Here she made an unfortunate mistake in interpreting the data for the main critical purpose of her article.

The soldiers' bonus is not part of "other money receipts" (O) but rather a part of "disposable money income" (Y). It is the main part of an item in the disposable money income category called "military pay, allotments, and pensions" [4, Vol. 11, p. xxix].

This would appear to alter completely the relationship of Professor Reid's main findings to the Bodkin results and to change the windfall interpretation of the O variable. Surely, fire and accident policy settlements are not windfall income, but rather a (partial) recovery of real assets previously lost. Likewise, inheritances are probably best considered as a long-anticipated increase in assets—not an increase in transitory income.

The discovery of this error probably does not affect whatever importance Professor Reid's secondary finding may have: ". . . the need, in any study of

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NEW REGULATORY FINANCE

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yield effects in Figure 6-2 can be imagined, substituting a skewness line for the dividend yield line.

California water utilities in the late 1990s and continuing in the 2000s provide a good example of skewness effects. Because of the asymmetry in the future water supply, there is a greater probability of downside returns to investors under adverse supply conditions, but essentially no probability of correspondingly large positive returns. That is, these water utilities' future profitability is constrained by both the regulatory process and by a negatively skewed water supply. Hence, measures of variability and covariability, such as standard deviation and beta, are likely to provide downward-biased estimates of the true risk relative to that of unregulated firms and other utilities.

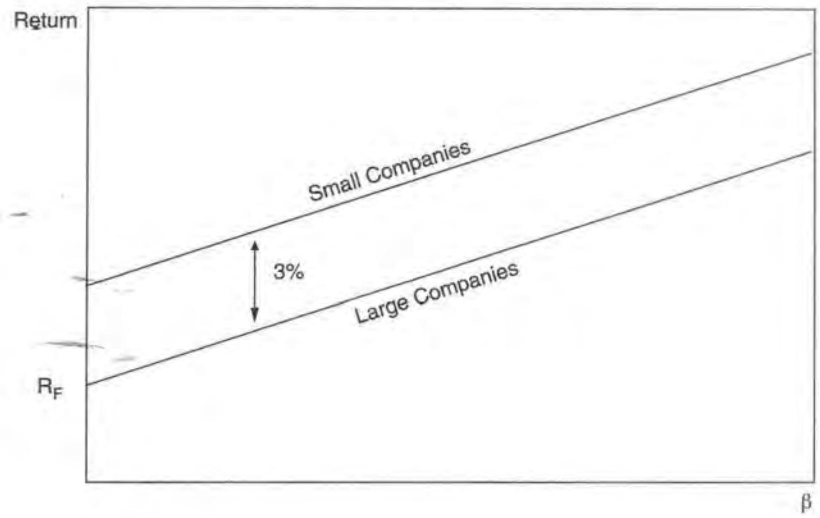
Another example is provided by some regulatory incentive plans where the risks of potential losses are borne exclusively by shareholders due to an absence of any return floor and the presence of a limit on the allowable recovery of cost increases. The benefits of added efficiencies and productivity gains achieved by the company over and above the allowed return are absorbed totally by ratepayers. Such lack of symmetry (''heads I win, tails you lose'') clearly increases risk and results in a deterioration of the regulatory climate and higher capital costs.

In both of these examples, the implication is that an additional risk premium must be added to the business-as-usual return on equity to compensate for the added risks. The lack of symmetry in investor returns must be considered. A risk premium sufficient to compensate investors for the limited upside returns/unlimited downside returns versus comparable risk companies and other utilities is required. To wit, in California's New Regulatory Framework designed to regulate large telecommunications companies, a 50 basis point increment was added to the benchmark rate of return on equity in order to compensate for the lack of symmetry in the plan.

Size Effect

Investment risk increases as company size diminishes, all else remaining constant. Small companies have very different returns than large ones, and on average they have been higher. The greater risk of small stocks does not fully account for their higher returns over many historical periods. The size phenomenon is well-documented in the finance literature. Empirical studies by Banz (1981) and Reinganum (1981A) have found that investors in small-capitalization stocks require higher returns than predicted by the standard CAPM. Reinganum (1981A) examined the relationship between the size of the firm and its P/E ratio, and found that small firms experienced average returns greater than those of large firms that were of equivalent systematic risk (beta). He found that small firms produce greater returns than could be explained by their risks. These results were confirmed in a separate test by

FIGURE 6-4
CAPM AND COMPANY SIZE



Banz (1981) who examined stock returns over the much longer 1936–1975 period, finding that stocks of small firms earned higher risk-adjusted abnormal returns than those of large firms. Fama and French (1992, 1993, 1997) find that company size and the reciprocal of the M/B ratio are significantly related to stock returns (cost of equity). The Fama-French asset pricing model is discussed later in this chapter.

The relationship between firm size and return cuts across the entire size spectrum but is most evident among smaller companies that have higher returns than larger ones on average. Ibbotson Associates' well-known historical return series publication covering the period 1926 to the present reinforces this evidence (Ibbotson Associates' *2005 Yearbook, Valuation Edition*). To illustrate, the Ibbotson data suggests that under SIC Code 49, *Electric, Gas & Sanitary Services*, the average return for that group over an almost 80-year period was 14.03% for the small-cap company group and 10.86% for the large-cap group, more than a 300 basis point difference. This is true for all industry groups. Overall, for the period 1926–2004, Ibbotson finds that the smaller companies have experienced returns that are not fully explainable by their higher betas, and that the excess return of that predicted by the CAPM increases as size decreases, suggesting that the cost of equity for small stocks is considerably larger than for large capitalization stocks. Ibbotson Associates provides estimates of the size premium required to be added to the basic CAPM cost of equity, shown in the following table. Figure 6-4 portrays the situation graphically.

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Chapter 7

Firm Size and Return

The Firm Size Phenomenon

One of the most remarkable discoveries of modern finance is the finding of a relationship between firm size and return.[1] On average, small companies have higher returns than large ones. Earlier chapters document this phenomenon for the smallest stocks on the New York Stock Exchange (NYSE). The relationship between firm size and return cuts across the entire size spectrum; it is not restricted to the smallest stocks. In this chapter, the returns across the entire range of firm size are examined.

Construction of the Size Decile Portfolios

The portfolios used in this chapter are those created by the Center for Research in Security Prices (CRSP) at the University of Chicago's Booth School of Business. CRSP has refined the methodology of creating size-based portfolios and has applied this methodology to the entire universe of NYSE/AMEX/NASDAQ-listed securities going back to 1926.

In 1993, CRSP changed the method used to construct these portfolios, thereby causing the return and index values in Table 7-2 and 7-3 to be significantly different from those reported in previous editions of the Yearbook. Previously, some eligible companies had been excluded or delayed from inclusion when the portfolios were reformed at the end of each calendar quarter. Also, while in prior editions of the Yearbook we used NYSE-listed securities only in the composition of size decile portfolios, starting with the 2001 edition we use the entire population of NYSE, NYSE AMEX, and NASDAQ-listed securities for use in the firm size chapter.

The New York Stock Exchange universe excludes closed-end mutual funds, preferred stocks, real estate investment trusts, foreign stocks, American Depository Receipts, unit investment trusts, and Americus Trusts. All companies on the NYSE are ranked by the combined market capitalization of all their eligible equity securities. The companies are then split into 10 equally populated groups or deciles. Eligible companies traded on the American Stock Exchange (AMEX) and the Nasdaq National Market (NASDAQ) are then assigned to the appropriate deciles according to their capitalization in relation to the NYSE breakpoints. The portfolios are rebalanced using closing prices for the last trading day of March, June, September, and December. Securities added during the quarter are assigned to the appropriate portfolio when two consecutive month-end prices are available. If the final NYSE price of a security that becomes delisted is a month-end price, then that month's return is included in the quarterly return of the portfolio. When a month-end NYSE price is missing, the month-end value is derived from merger terms, quotations on regional exchanges, and other sources. If a month-end value is not available, the last available daily price is used.

Base security returns are monthly holding period returns. All distributions are added to the month-end prices. Appropriate adjustments are made to prices to account for stock splits and dividends. The return on a portfolio for one month is calculated as the weighted average of the returns for the individual stocks in the portfolio. Annual portfolio returns are calculated by compounding the monthly portfolio returns.

Aspects of the Firm Size Effect

The firm size phenomenon is remarkable in several ways. First, the greater risk of small stocks does not, in the context of the Capital Asset Pricing Model, fully account for their higher returns over the long term. In the CAPM, only systematic, or beta risk, is rewarded. Small company stocks have had returns in excess of those implied by the betas of small stocks. Secondly, the calendar annual return differences between small and large companies are serially correlated. This suggests that past annual returns may be of some value in predicting future annual returns. Such serial correlation, or autocorrelation, is practically unknown in the market for large stocks and in most other capital markets.

In addition, the firm size effect is seasonal. For example, small company stocks outperformed large company stocks in the month of January in a large majority of the years. Again, such predictability is surprising and suspicious in the light of modern capital market theory. These three aspects of the firm size effect (long-term returns in excess of risk, serial correlation and seasonality) will be analyzed after the data are presented.

Presentation of the Decile Data

Summary statistics of annual returns of the 10 deciles and size groupings from 1926–2009 are presented in Table 7-1. Note from this exhibit that the average return tends to increase as one moves from the largest decile to the smallest. (Because securities are ranked quarterly, returns on the ninth and tenth deciles are different than those suggested by the small company stock index presented in earlier chapters. A detailed methodology for the small company stock index is included in Chapter 3.) The total risk, or standard deviation of annual returns, also increases with decreasing firm size. The serial correlations of returns are near zero for all but the smallest decile.

Table 7-1: Size-Decile Portfolios of the NYSE/AMEX/NASDAQ Summary Statistics of Annual Returns

Decile	Geometric Mean	Arithmetic Mean	Standard Deviation	Serial Correlation
1-Largest	9.1	10.9	19.4	0.07
2	10.4	12.8	22.4	0.01
3	10.7	13.4	23.9	-0.04
4	10.7	13.8	26.2	-0.03
5	11.3	14.6	27.0	-0.04
6	11.2	14.8	27.6	0.02
7	11.2	15.2	29.8	0.00
8	11.4	16.3	34.4	0.04
9	11.5	17.0	36.7	0.04
10 - Smallest	13.1	20.9	45.2	0.14
Mid Cap	10.9	13.7	25.0	-0.04
Low Cap	11.3	15.2	29.4	0.02
Micro	12.1	18.2	39.2	0.07
NYSE/AMEX/ NASDAQ Total Value Weighted Index	9.6	11.6	20.5	0.01

Data from 1926–2009. Source: Morningstar and CRSP. Calculated (or Derived) based on data from CRSP US Stock Database and CRSP US Indices Database ©2010 Center for Research in Security Prices (CRSP®), The University of Chicago Booth School of Business. Used with permission.

Results are for quarterly re-ranking for the deciles. The small company stock summary statistics presented in earlier chapters comprise a re-ranking of the portfolios every five years prior to 1982.

Table 7-2 is a year-by-year history of the returns for the different size categories. Table 7-3 shows the growth of $1.00 invested in each of the categories at year-end 1925.

The sheer magnitude of the size effect in some years is noteworthy. While the largest stocks actually declined in 2001, the smallest stocks rose more than 30 percent. A more extreme case occurred in the depression-recovery year of 1933, when the difference between the first and tenth decile returns was far more substantial. The divergence in the performance of small and large company stocks is evident. In 29 of the 84 years since 1926, the difference between the total returns of the largest stocks (decile 1) and the smallest stocks (decile 10) has been greater than 25 percent.

Table 7-2

Size-Decile Portfolios of the NYSE/AMEX/NASDAQ
Annual Returns

1926–1970

	Decile 1	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10
1926	0.1358	0.0637	0.0247	0.0209	-0.0236	0.0522	-0.0143	-0.1085	-0.0815	-0.0526
1927	0.3436	0.3051	0.3102	0.3887	0.3427	0.2553	0.3468	0.2834	0.2567	0.2606
1928	0.3939	0.3748	0.3844	0.3471	0.5516	0.2716	0.3485	0.3156	0.3983	0.6894
1929	-0.1094	-0.0796	-0.2195	-0.3432	-0.2510	-0.4037	-0.3726	-0.4019	-0.4976	-0.5266
1930	-0.2456	-0.3749	-0.3527	-0.3488	-0.3578	-0.3639	-0.3624	-0.4931	-0.4463	-0.4834
1931	-0.4141	-0.5114	-0.4596	-0.4609	-0.4695	-0.5174	-0.4881	-0.4928	-0.5029	-0.4942
1932	-0.1096	0.0252	-0.0374	-0.1264	-0.1378	0.0726	-0.1440	0.0242	-0.0093	0.3981
1933	0.4599	0.7625	1.0087	1.1243	0.9505	1.0247	1.1045	1.7322	1.7477	2.1845
1934	0.0208	0.0583	0.0852	0.1845	0.0929	0.1951	0.1434	0.3076	0.2156	0.3489
1935	0.4170	0.5630	0.3705	0.3753	0.6521	0.5091	0.6677	0.6459	0.5849	0.8226
1936	0.2990	0.3436	0.2736	0.4170	0.4952	0.4928	0.5413	0.5028	0.8772	0.8546
1937	-0.3189	-0.3699	-0.3808	-0.4371	-0.4852	-0.4664	-0.4930	-0.5278	-0.5231	-0.5645
1938	0.2501	0.3401	0.3423	0.3512	0.5040	0.4189	0.3574	0.4344	0.3385	0.0540
1939	0.0480	-0.0387	-0.0279	0.0042	0.0157	0.0603	0.0482	-0.0425	-0.0526	0.1737
1940	-0.0702	-0.0884	-0.0837	-0.0404	-0.0079	-0.0580	-0.0574	-0.0634	-0.0491	-0.3114
1941	-0.1069	-0.0778	-0.0590	-0.0984	-0.1197	-0.0990	-0.0890	-0.0886	-0.1253	-0.1798
1942	0.1337	0.2365	0.2026	0.2031	0.2097	0.2463	0.2912	0.2971	0.4429	0.8021
1943	0.2350	0.3526	0.3343	0.4049	0.4949	0.4129	0.7226	0.7146	0.8725	1.3764
1944	0.1719	0.2539	0.2294	0.3308	0.4003	0.4405	0.3841	0.4886	0.5655	0.7003
1945	0.2950	0.4758	0.5448	0.6365	0.5341	0.6106	0.6509	0.6895	0.7690	0.9554
1946	-0.0446	-0.0439	-0.0781	-0.1268	-0.1022	-0.0617	-0.1485	-0.1535	-0.0972	-0.1833
1947	0.0555	0.0076	-0.0020	0.0207	0.0342	-0.0335	-0.0217	-0.0323	-0.0356	-0.0088
1948	0.0371	0.0016	0.0253	-0.0207	-0.0253	-0.0345	-0.0329	-0.0659	-0.0741	-0.0520
1949	0.1858	0.2521	0.2595	0.1953	0.1861	0.2329	0.2177	0.1652	0.1979	0.2489
1950	0.2881	0.2892	0.2672	0.3137	0.3703	0.3387	0.3786	0.3995	0.4132	0.5514
1951	0.2141	0.2286	0.2116	0.1663	0.1442	0.1372	0.1811	0.1511	0.1125	0.0685
1952	0.1428	0.1293	0.1216	0.1190	0.1107	0.1010	0.1039	0.0768	0.0852	0.0230
1953	0.0115	0.0169	0.0033	-0.0136	-0.0293	-0.0095	-0.0241	-0.0772	-0.0494	-0.0818
1954	0.4833	0.4825	0.5892	0.5081	0.5673	0.5955	0.5738	0.5287	0.6373	0.6863
1955	0.2846	0.1877	0.1834	0.1932	0.1771	0.2265	0.1843	0.2023	0.2053	0.2553
1956	0.0794	0.1108	0.0741	0.0902	0.0805	0.0594	0.0830	0.0522	0.0589	-0.0165
1957	-0.0932	-0.0869	-0.1285	-0.1079	-0.1384	-0.1821	-0.1677	-0.1855	-0.1424	-0.1679
1958	0.4071	0.4981	0.5406	0.5964	0.5583	0.5627	0.6814	0.6527	0.7144	0.6975
1959	0.1236	0.0967	0.1363	0.1524	0.1994	0.1516	0.1987	0.1799	0.2011	0.1542
1960	0.0037	0.0548	0.0482	0.0128	-0.0165	-0.0087	-0.0586	-0.0511	-0.0380	-0.0786
1961	0.2627	0.2710	0.2898	0.2933	0.2853	0.2699	0.3043	0.3377	0.3030	0.3202
1962	-0.0878	-0.0959	-0.1194	-0.1296	-0.1638	-0.1793	-0.1640	-0.1476	-0.1701	-0.1460
1963	0.2249	0.2141	0.1647	0.1712	0.1273	0.1853	0.1782	0.1997	0.1280	0.1117
1964	0.1599	0.1428	0.1997	0.1625	0.1623	0.1666	0.1597	0.1714	0.1532	0.2094
1965	0.0893	0.1925	0.2483	0.2425	0.3217	0.3776	0.3373	0.3190	0.3194	0.4315
1966	-0.1027	-0.0574	-0.0507	-0.0623	-0.0721	-0.0452	-0.0955	-0.0864	-0.0589	-0.1008
1967	0.2197	0.2079	0.3169	0.4564	0.5145	0.5343	0.6472	0.8133	0.9064	1.1416
1968	0.0753	0.1654	0.1979	0.1829	0.2759	0.3047	0.2673	0.4047	0.3711	0.6136
1969	-0.0584	-0.1295	-0.1172	-0.1662	-0.1808	-0.1871	-0.2445	-0.2471	-0.3158	-0.3290
1970	0.0231	0.0182	0.0330	-0.0699	-0.0601	-0.0593	-0.0973	-0.1614	-0.1526	-0.1785

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Table 7-2 (Continued)

Size-Decile Portfolios of the NYSE/AMEX/NASDAQ

Annual Returns

1971–2009

	Decile 1	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10
1971	0.1484	0.1328	0.2011	0.2472	0.1890	0.2244	0.2018	0.1735	0.1647	0.1853
1972	0.2212	0.1278	0.0938	0.0881	0.0863	0.0695	0.0632	0.0205	-0.0229	-0.0057
1973	-0.1274	-0.2266	-0.2278	-0.2680	-0.3217	-0.3191	-0.3702	-0.3534	-0.3897	-0.4203
1974	-0.2803	-0.2441	-0.2458	-0.2834	-0.2167	-0.2694	-0.2558	-0.2423	-0.2635	-0.2715
1975	0.3169	0.4573	0.5363	0.6168	0.5966	0.5675	0.6326	0.6579	0.6649	0.7579
1976	0.2073	0.3045	0.3811	0.4008	0.4363	0.4808	0.5018	0.5690	0.5101	0.5516
1977	-0.0884	-0.0367	0.0109	0.0376	0.1126	0.1408	0.1754	0.2261	0.2022	0.2310
1978	0.0637	0.0229	0.1084	0.0974	0.1207	0.1637	0.1705	0.1632	0.1605	0.2815
1979	0.1519	0.2871	0.3061	0.3516	0.3557	0.4888	0.4206	0.4638	0.4594	0.4158
1980	0.3275	0.3442	0.3186	0.3043	0.3193	0.3141	0.3623	0.3233	0.3823	0.3071
1981	-0.0833	0.0059	0.0372	0.0403	0.0484	0.0677	-0.0040	0.0055	0.0802	0.0856
1982	0.1964	0.1749	0.2081	0.2566	0.3076	0.2940	0.2919	0.2955	0.2608	0.2855
1983	0.2057	0.1686	0.2662	0.2633	0.2626	0.2589	0.2727	0.3721	0.3130	0.3690
1984	0.0840	0.0770	0.0253	-0.0458	-0.0269	0.0248	-0.0426	-0.0745	-0.0896	-0.1951
1985	0.3137	0.3770	0.2910	0.3390	0.3115	0.3097	0.3255	0.3651	0.3077	0.2582
1986	0.1801	0.1816	0.1628	0.1732	0.1512	0.0874	0.1248	0.0387	0.0570	0.0041
1987	0.0504	0.0037	0.0393	0.0170	-0.0382	-0.0508	-0.0861	-0.0808	-0.1262	-0.1492
1988	0.1486	0.1982	0.2126	0.2237	0.2138	0.2339	0.2394	0.2854	0.2285	0.2105
1989	0.3295	0.3008	0.2629	0.2308	0.2423	0.2107	0.1785	0.1788	0.1058	0.0550
1990	-0.0088	-0.0853	-0.1015	-0.0875	-0.1409	-0.1849	-0.1532	-0.1979	-0.2460	-0.3128
1991	0.3039	0.3463	0.4140	0.3883	0.4811	0.5326	0.4421	0.4707	0.5066	0.4807
1992	0.0474	0.1577	0.1387	0.1249	0.2613	0.1878	0.1920	0.1287	0.2495	0.3398
1993	0.0732	0.1319	0.1614	0.1562	0.1694	0.1726	0.1900	0.1853	0.1658	0.2558
1994	0.0174	-0.0174	-0.0423	-0.0098	-0.0166	0.0034	-0.0252	-0.0308	-0.0309	-0.0298
1995	0.3940	0.3527	0.3533	0.3276	0.3324	0.2692	0.3264	0.2935	0.3500	0.3047
1996	0.2375	0.1962	0.1714	0.1883	0.1366	0.1737	0.1965	0.1720	0.2064	0.1722
1997	0.3486	0.3012	0.2512	0.2610	0.1566	0.2864	0.3003	0.2538	0.2554	0.2204
1998	0.3515	0.1272	0.0758	0.0724	0.0054	0.0116	-0.0090	0.0098	-0.0503	-0.1155
1999	0.2450	0.2018	0.3404	0.2966	0.2595	0.3492	0.2570	0.3886	0.3430	0.2809
2000	-0.1359	-0.0030	-0.0620	-0.0997	-0.0710	-0.1028	-0.1068	-0.1300	-0.1331	-0.1291
2001	-0.1529	-0.0881	-0.0411	-0.0096	-0.0214	0.0952	0.1226	0.2111	0.3168	0.3676
2002	-0.2246	-0.1736	-0.1934	-0.1771	-0.1778	-0.2122	-0.2297	-0.1994	-0.1870	-0.0550
2003	0.2568	0.3738	0.4029	0.4438	0.4090	0.4877	0.5074	0.5761	0.6783	0.9245
2004	0.0794	0.2013	0.1796	0.1874	0.1734	0.2206	0.1904	0.2196	0.1518	0.1857
2005	0.0371	0.1221	0.1237	0.1059	0.1011	0.0306	0.1058	0.0753	0.0216	0.0591
2006	0.1561	0.1559	0.1453	0.1164	0.1557	0.1504	0.1627	0.1761	0.1713	0.1948
2007	0.0718	0.0747	0.0362	0.0436	0.0785	0.0498	-0.0159	-0.0559	-0.0647	-0.0992
2008	-0.3508	-0.4190	-0.4035	-0.3679	-0.3539	-0.3998	-0.3620	-0.3561	-0.3683	-0.4733
2009	0.2256	0.3825	0.3814	0.4479	0.4471	0.4204	0.4340	0.4773	0.4957	0.8113

Source: Morningstar and CRSP. Calculated (or Derived) based on data from CRSP US Stock Database and CRSP US Indices Database

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Table 7-3

Size-Decile Portfolios of the NYSE/AMEX/NASDAQ
Year-End Index Values

1925–1970

	Decile 1	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10
1925	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1926	1.136	1.064	1.025	1.021	0.976	1.052	0.986	0.892	0.919	0.947
1927	1.526	1.388	1.343	1.418	1.311	1.321	1.328	1.144	1.154	1.194
1928	2.127	1.909	1.859	1.910	2.034	1.679	1.790	1.505	1.614	2.018
1929	1.895	1.757	1.451	1.254	1.524	1.002	1.123	0.900	0.811	0.955
1930	1.429	1.098	0.939	0.817	0.978	0.637	0.716	0.456	0.449	0.493
1931	0.837	0.536	0.507	0.440	0.519	0.307	0.367	0.231	0.223	0.250
1932	0.746	0.550	0.488	0.385	0.447	0.330	0.314	0.237	0.221	0.349
1933	1.088	0.969	0.981	0.817	0.873	0.668	0.660	0.648	0.608	1.111
1934	1.111	1.026	1.065	0.968	0.954	0.798	0.755	0.847	0.739	1.499
1935	1.574	1.604	1.459	1.331	1.576	1.204	1.260	1.394	1.171	2.732
1936	2.045	2.155	1.858	1.886	2.356	1.798	1.941	2.094	2.197	5.067
1937	1.393	1.358	1.151	1.062	1.213	0.959	0.984	0.989	1.048	2.206
1938	1.741	1.819	1.545	1.434	1.825	1.361	1.336	1.418	1.403	2.326
1939	1.825	1.749	1.502	1.440	1.853	1.443	1.401	1.358	1.329	2.729
1940	1.697	1.594	1.376	1.382	1.839	1.359	1.320	1.272	1.264	1.880
1941	1.515	1.470	1.295	1.246	1.619	1.225	1.203	1.159	1.105	1.542
1942	1.718	1.818	1.557	1.499	1.958	1.526	1.553	1.504	1.595	2.778
1943	2.122	2.459	2.078	2.106	2.927	2.157	2.675	2.579	2.987	6.602
1944	2.487	3.083	2.554	2.803	4.099	3.107	3.702	3.838	4.675	11.225
1945	3.220	4.550	3.946	4.587	6.288	5.003	6.112	6.485	8.271	21.950
1946	3.077	4.350	3.638	4.005	5.645	4.695	5.205	5.490	7.467	17.926
1947	3.247	4.383	3.630	4.088	5.838	4.538	5.092	5.312	7.201	17.768
1948	3.368	4.390	3.722	4.004	5.691	4.381	4.924	4.962	6.668	16.844
1949	3.994	5.496	4.688	4.786	6.750	5.401	5.997	5.782	7.988	21.036
1950	5.144	7.086	5.940	6.287	9.249	7.231	8.267	8.092	11.288	32.636
1951	6.245	8.705	7.197	7.333	10.583	8.223	9.764	9.314	12.558	34.871
1952	7.137	9.831	8.072	8.205	11.755	9.054	10.778	10.030	13.629	35.673
1953	7.219	9.997	8.099	8.093	11.410	8.968	10.518	9.256	12.956	32.754
1954	10.708	14.820	12.871	12.205	17.884	14.309	16.553	14.149	21.213	55.233
1955	13.755	17.602	15.232	14.564	21.051	17.550	19.604	17.011	25.568	69.335
1956	14.847	19.553	16.361	15.878	22.747	18.593	21.230	17.899	27.076	68.193
1957	13.464	17.854	14.258	14.165	19.599	15.207	17.670	14.580	23.221	56.742
1958	18.945	26.748	21.967	22.613	30.541	23.765	29.710	24.096	39.810	96.319
1959	21.287	29.335	24.961	26.060	36.630	27.368	35.613	28.430	47.817	111.172
1960	21.366	30.943	26.164	26.393	36.025	27.128	33.525	26.978	46.000	102.433
1961	26.979	39.328	33.747	34.135	46.302	34.449	43.725	36.089	59.940	135.228
1962	24.610	35.558	29.716	29.713	38.719	28.274	36.555	30.761	49.742	115.483
1963	30.143	43.170	34.612	34.799	43.648	33.514	43.068	36.902	56.106	128.379
1964	34.962	49.335	41.524	40.455	50.734	39.098	49.946	43.225	64.702	155.258
1965	38.083	58.833	51.832	50.265	67.054	53.861	66.795	57.014	85.371	222.249
1966	34.171	55.455	49.202	47.136	62.222	51.426	60.414	52.090	80.339	199.850
1967	41.678	66.984	64.794	68.648	94.237	78.902	99.515	94.457	153.162	427.993
1968	44.817	78.066	77.615	81.201	120.233	102.941	126.119	132.679	210.006	690.608
1969	42.198	67.956	68.519	67.709	98.500	83.685	95.279	99.892	143.692	463.384
1970	43.174	69.190	70.783	62.976	92.584	78.721	86.008	83.772	121.764	380.688

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Table 7-3 (Continued)

Size-Decile Portfolios of the NYSE/AMEX/NASDAQ

Year-End Index Values

1971–2009

	Decile 1	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10
1971	49.582	78.380	85.019	78.544	110.084	96.390	103.365	98.304	141.813	451.246
1972	60.549	88.400	92.996	85.463	119.582	103.087	109.900	100.320	138.561	448.673
1973	52.838	68.365	71.812	62.555	81.109	70.189	69.218	64.866	84.561	260.087
1974	38.026	51.679	54.159	44.825	63.532	51.282	51.514	49.149	62.283	189.465
1975	50.075	75.310	83.204	72.474	101.434	80.386	84.101	81.481	103.694	333.057
1976	60.457	98.244	114.912	101.522	145.692	119.033	126.303	127.842	156.591	516.784
1977	55.114	94.641	116.161	105.340	162.096	135.795	148.457	156.753	188.254	636.174
1978	58.623	96.810	128.757	115.602	181.663	158.018	173.769	182.335	218.469	815.247
1979	67.529	124.608	168.171	156.247	246.280	235.265	246.858	266.896	318.844	1154.203
1980	89.642	167.495	221.752	203.790	324.928	309.150	336.293	353.172	440.733	1508.688
1981	82.171	168.486	230.000	212.012	340.658	330.086	334.956	355.111	476.060	1637.806
1982	98.309	197.961	277.862	266.417	445.453	427.119	432.737	460.051	600.204	2105.324
1983	118.536	231.336	351.823	336.573	562.436	537.709	550.745	631.240	788.094	2882.280
1984	128.496	249.156	360.708	321.152	547.328	551.050	527.300	584.194	717.520	2319.945
1985	168.799	343.079	465.674	430.017	717.830	721.717	698.952	797.490	938.290	2918.990
1986	199.203	405.372	541.507	504.509	826.375	784.763	786.180	828.328	991.802	2930.952
1987	209.237	406.864	562.767	513.077	794.795	744.859	718.467	761.390	866.642	2493.635
1988	240.323	487.505	682.400	627.867	964.734	919.100	890.492	978.714	1064.661	3018.511
1989	319.517	634.137	861.817	772.773	1198.484	1112.769	1049.459	1153.680	1177.323	3184.493
1990	316.696	580.036	774.312	705.157	1029.606	906.966	888.732	925.311	887.722	2188.421
1991	412.940	780.876	1094.907	978.935	1524.998	1390.054	1281.599	1360.883	1337.401	3240.359
1992	432.528	904.053	1246.792	1101.211	1923.542	1651.164	1527.706	1535.982	1671.149	4341.570
1993	464.200	1023.276	1448.020	1273.167	2249.452	1936.090	1818.024	1820.666	1948.174	5452.331
1994	472.281	1005.505	1386.834	1260.636	2212.120	1942.657	1772.157	1764.589	1887.894	5289.982
1995	658.354	1360.139	1876.818	1673.588	2947.424	2465.680	2350.677	2282.491	2548.704	6902.067
1996	814.721	1626.941	2198.551	1988.737	3350.013	2893.890	2812.472	2675.160	3074.639	8090.904
1997	1098.728	2116.959	2750.825	2507.894	3874.475	3722.689	3657.082	3354.002	3859.974	9874.270
1998	1484.945	2386.193	2959.244	2689.548	3895.300	3765.924	3624.257	3386.763	3665.743	8733.362
1999	1848.820	2867.706	3966.538	3487.400	4906.053	5081.009	4555.730	4702.836	4923.248	11186.740
2000	1597.583	2859.238	3720.436	3139.817	4557.657	4558.894	4069.294	4091.354	4267.999	9742.548
2001	1353.311	2607.248	3567.403	3109.557	4460.000	4993.107	4568.166	4954.981	5620.110	13323.510
2002	1049.390	2154.621	2877.420	2558.865	3667.100	3933.746	3518.738	3966.994	4569.073	12590.514
2003	1318.847	2960.002	4036.603	3694.589	5166.919	5852.310	5304.257	6252.256	7668.054	24230.073
2004	1423.611	3555.873	4761.488	4386.977	6062.734	7143.115	6314.087	7625.189	8832.170	28730.582
2005	1476.498	3990.023	5350.498	4851.512	6675.503	7361.863	6982.197	8199.603	9022.881	30428.623
2006	1707.026	4612.246	6128.086	5416.218	7715.172	8469.343	8118.430	9643.387	10568.327	36357.297
2007	1829.638	4957.001	6349.776	5652.208	8320.755	8890.975	7989.212	9104.422	9884.069	32751.548
2008	1187.842	2879.989	3787.667	3572.535	5376.004	5336.079	5097.215	5862.638	6243.843	17249.292
2009	1455.799	3981.479	5232.471	5172.596	7779.671	7579.329	7309.405	8660.930	9338.614	31242.844

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Table 7-4

Size-Decile Portfolios of the NYSE/AMEX/NASDAQ
Mid-, Low-, Micro-, and Total Capitalization Returns
and Index Values

1925–1965

	Total Return				Index Value			
Year	Mid-Cap Stocks	Low-Cap Stocks	Micro-Cap Stocks	Total Value Weighted NYSE/ AMEX/ NASDAQ	Mid-Cap Stocks	Low-Cap Stocks	Micro-Cap Stocks	Total Value Weighted NYSE/ AMEX/ NASDAQ
1925					1.000	1.000	1.000	1.000
1926	0.0144	-0.0042	-0.0721	0.0924	1.014	0.996	0.928	1.092
1927	0.3394	0.2912	0.2576	0.3339	1.359	1.286	1.167	1.457
1928	0.4040	0.3057	0.4685	0.3886	1.908	1.679	1.714	2.023
1929	-0.2623	-0.3925	-0.5044	-0.1455	1.407	1.020	0.849	1.729
1930	-0.3521	-0.3898	-0.4555	-0.2854	0.912	0.622	0.462	1.236
1931	-0.4615	-0.5040	-0.5028	-0.4354	0.491	0.309	0.230	0.698
1932	-0.0794	-0.0077	0.0890	-0.0869	0.452	0.306	0.250	0.637
1933	1.0294	1.1752	1.8695	0.5709	0.917	0.666	0.718	1.001
1934	0.1159	0.1990	0.2509	0.0428	1.024	0.799	0.899	1.043
1935	0.4181	0.5860	0.6484	0.4432	1.452	1.267	1.481	1.506
1936	0.3594	0.5101	0.8749	0.3226	1.973	1.913	2.777	1.992
1937	-0.4200	-0.4874	-0.5340	-0.3469	1.145	0.981	1.294	1.301
1938	0.3756	0.4029	0.2625	0.2809	1.575	1.376	1.634	1.666
1939	-0.0095	0.0360	0.0021	0.0286	1.560	1.425	1.637	1.714
1940	-0.0553	-0.0588	-0.1236	-0.0708	1.473	1.342	1.435	1.592
1941	-0.0835	-0.0934	-0.1373	-0.1003	1.350	1.216	1.238	1.433
1942	0.2042	0.2705	0.5247	0.1604	1.626	1.545	1.888	1.662
1943	0.3868	0.5728	1.0007	0.2838	2.255	2.430	3.776	2.134
1944	0.2950	0.4323	0.6040	0.2131	2.920	3.481	6.057	2.589
1945	0.5704	0.6418	0.8265	0.3807	4.586	5.715	11.064	3.575
1946	-0.0986	-0.1126	-0.1260	-0.0586	4.134	5.072	9.669	3.365
1947	0.0128	-0.0296	-0.0266	0.0358	4.187	4.922	9.412	3.486
1948	0.0000	-0.0413	-0.0660	0.0211	4.187	4.719	8.790	3.559
1949	0.2243	0.2127	0.2149	0.2022	5.126	5.722	10.679	4.279
1950	0.3027	0.3655	0.4591	0.2961	6.677	7.814	15.583	5.546
1951	0.1830	0.1546	0.0977	0.2068	7.900	9.022	17.105	6.693
1952	0.1186	0.0966	0.0647	0.1342	8.836	9.894	18.212	7.591
1953	-0.0084	-0.0290	-0.0598	0.0067	8.762	9.607	17.122	7.642
1954	0.5604	0.5747	0.6523	0.4998	13.673	15.128	28.291	11.461
1955	0.1850	0.2078	0.2211	0.2521	16.202	18.273	34.546	14.351
1956	0.0803	0.0653	0.0346	0.0826	17.503	19.466	35.741	15.537
1957	-0.1242	-0.1783	-0.1505	-0.1005	15.329	15.996	30.361	13.975
1958	0.5611	0.6186	0.7092	0.4502	23.931	25.891	51.894	20.267
1959	0.1536	0.1726	0.1868	0.1267	27.608	30.359	61.588	22.835
1960	0.0243	-0.0339	-0.0500	0.0116	28.279	29.331	58.506	23.100
1961	0.2899	0.2951	0.3084	0.2695	36.477	37.986	76.547	29.324
1962	-0.1315	-0.1683	-0.1650	-0.1018	31.682	31.593	63.920	26.340
1963	0.1593	0.1867	0.1193	0.2098	36.731	37.490	71.547	31.866
1964	0.1813	0.1652	0.1834	0.1613	43.392	43.685	84.672	37.004
1965	0.2608	0.3499	0.3798	0.1446	54.708	58.971	116.833	42.356

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Table 7-4 (Continued)
Size-Decile Portfolios of the NYSE/AMEX/NASDAQ
Mid-, Low-, Micro-, and Total Capitalization Returns
and Index Values

1966–2009

Year	Total Return: Mid-Cap Stocks	Total Return: Low-Cap Stocks	Total Return: Micro-Cap Stocks	Total Return: Total Value Weighted NYSE/AMEX/NASDAQ	Index Value: Mid-Cap Stocks	Index Value: Low-Cap Stocks	Index Value: Micro-Cap Stocks	Index Value: Total Value Weighted NYSE/AMEX/NASDAQ
1966	-0.0586	-0.0710	-0.0825	-0.0874	51.502	54.785	107.189	38.654
1966	-0.0586	-0.0710	-0.0825	-0.0874	51.502	54.785	107.189	38.654
1967	0.3994	0.6387	1.0344	0.2874	72.070	89.779	218.063	49.763
1968	0.2108	0.3182	0.5015	0.1414	87.261	118.343	327.422	56.800
1969	-0.1469	-0.2216	-0.3236	-0.1091	74.445	92.116	221.456	50.601
1970	-0.0201	-0.0987	-0.1681	0.0000	72.947	83.027	184.224	50.602
1971	0.2123	0.2032	0.1767	0.1615	88.437	99.899	216.771	58.772
1972	0.0906	0.0558	-0.0138	0.1684	96.451	105.476	213.778	68.668
1973	-0.2594	-0.3435	-0.4078	-0.1806	71.432	69.247	126.594	56.263
1974	-0.2513	-0.2587	-0.2676	-0.2704	53.483	51.335	92.715	41.051
1975	0.5709	0.6092	0.7150	0.3875	84.016	82.609	159.003	56.960
1976	0.3979	0.5074	0.5335	0.2676	117.447	124.528	243.838	72.203
1977	0.0385	0.1708	0.2177	-0.0426	121.973	145.792	296.917	69.128
1978	0.1075	0.1663	0.2245	0.0749	135.081	170.033	363.563	74.303
1979	0.3298	0.4626	0.4369	0.2262	179.631	248.690	522.404	91.113
1980	0.3144	0.3310	0.3464	0.3281	236.104	330.995	703.379	121.011
1981	0.0409	0.0305	0.0818	-0.0365	245.761	341.091	760.892	116.596
1982	0.2443	0.2939	0.2723	0.2100	305.799	441.333	968.104	141.082
1983	0.2644	0.2882	0.3410	0.2198	386.645	568.525	1298.236	172.086
1984	-0.0103	-0.0224	-0.1403	0.0451	382.659	555.814	1116.057	179.849
1985	0.3115	0.3283	0.2833	0.3217	501.840	738.313	1432.191	237.703
1986	0.1637	0.0877	0.0320	0.1619	583.976	803.032	1478.083	276.188
1987	0.0130	-0.0689	-0.1381	0.0167	591.583	747.687	1273.886	280.801
1988	0.2167	0.2476	0.2192	0.1803	719.777	932.794	1553.116	331.423
1989	0.2479	0.1923	0.0815	0.2886	898.199	1112.149	1679.745	427.084
1990	-0.1053	-0.1779	-0.2744	-0.0596	803.585	914.340	1218.755	401.636
1991	0.4191	0.4865	0.5005	0.3467	1140.359	1359.121	1828.732	540.871
1992	0.1612	0.1738	0.2814	0.0980	1324.133	1595.389	2343.256	593.865
1993	0.1627	0.1824	0.2010	0.1114	1539.505	1886.336	2814.214	660.004
1994	-0.0263	-0.0152	-0.0314	-0.0006	1499.087	1857.587	2725.976	659.604
1995	0.3404	0.2947	0.3320	0.3679	2009.376	2405.034	3630.877	902.300
1996	0.1685	0.1804	0.1926	0.2135	2348.036	2838.819	4330.072	1094.950
1997	0.2329	0.2804	0.2402	0.3140	2894.819	3634.907	5370.151	1438.759
1998	0.0591	0.0051	-0.0817	0.2429	3065.774	3653.462	4931.268	1788.242
1999	0.3107	0.3290	0.3165	0.2527	4018.318	4855.359	6491.852	2240.054
2000	-0.0755	-0.1100	-0.1302	-0.1141	3714.844	4321.148	5646.670	1984.392
2001	-0.0280	0.1324	0.3399	-0.1115	3610.715	4893.280	7566.103	1763.231
2002	-0.1850	-0.2158	-0.1386	-0.2115	2942.738	3837.458	6517.072	1390.376
2003	0.4161	0.5173	0.7786	0.3161	4167.126	5822.500	11591.112	1829.916
2004	0.1807	0.2110	0.1670	0.1197	4919.965	7051.111	13526.855	2048.888
2005	0.1136	0.0677	0.0366	0.0616	5478.654	7528.428	14021.280	2175.152
2006	0.1387	0.1617	0.1810	0.1547	6238.765	8745.681	16558.950	2511.620
2007	0.0474	-0.0012	-0.0794	0.0583	6534.381	8734.751	15244.556	2658.067
2008	-0.3814	-0.3757	-0.4145	-0.3669	4041.981	5453.539	8925.741	1682.704
2009	0.4179	0.4405	0.6067	0.2881	5731.069	7855.944	14340.987	2167.571

Source: Morningstar and CRSP. Calculated (or Derived) based on data from CRSP US Stock Database and CRSP US Indices Database
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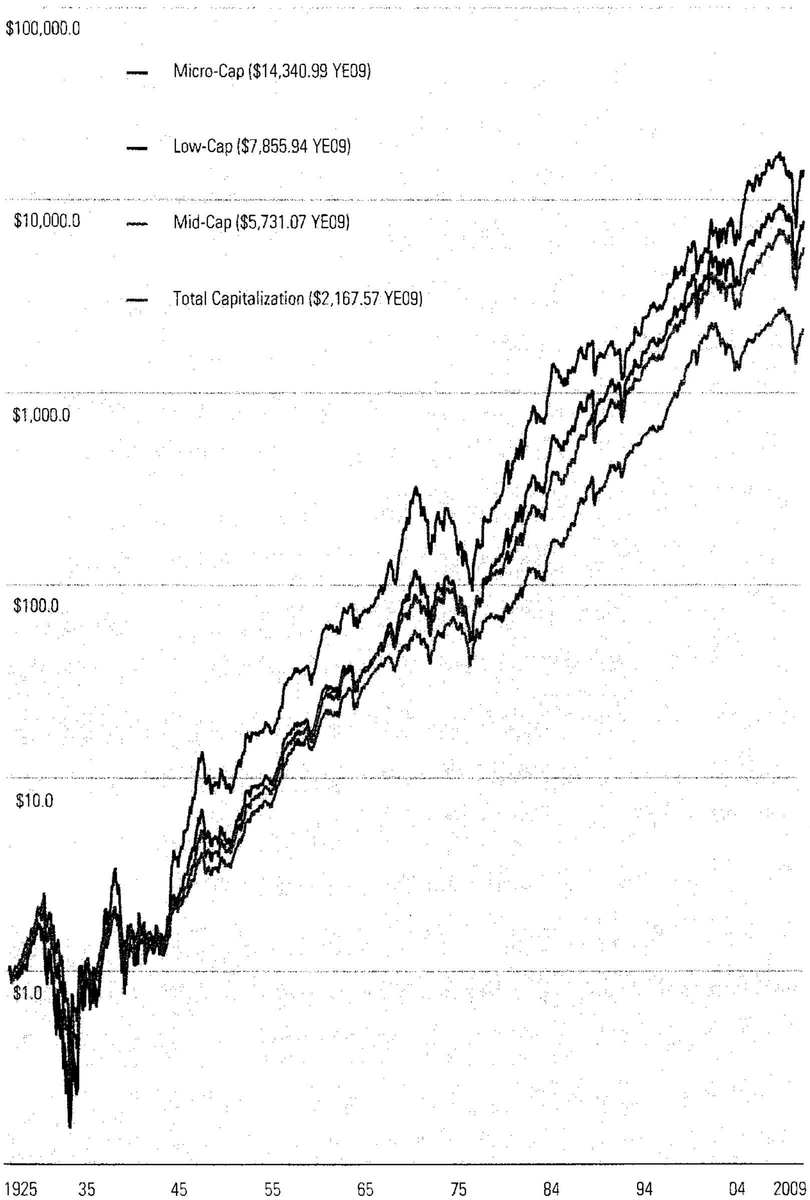
Graph 7-1: Size-Decile Portfolios of the NYSE/AMEX/NASDAQ
Wealth Indices of Investments in Mid-, Low-, Micro-, and Total Capitalization Stocks
Index (Year-End 1925 = $1.00)



Data from 1925–2009.

In Table 7-4, the decile returns and index values of the NYSE/AMEX/NASDAQ population are broken down into mid-cap, low-cap, and micro-cap stocks. Mid-cap stocks are defined here as the aggregate of deciles 3–5. Based on the most recent data, as shown in the bottom section of Table 7-5, companies within this mid-cap range have market capitalizations at or below $5,936,147,000 but greater than $1,600,169,000. Low-cap stocks include deciles 6–8, and currently include all companies in the NYSE/AMEX/NASDAQ with market capitalizations at or below $1,600,169,000 but greater than $431,256,000. Micro-cap stocks include deciles 9–10, and include companies with market capitalizations at or below $431,256,000. The returns and index values of the entire NYSE/AMEX/NASDAQ population are also included. All returns presented are value-weighted based on the market capitalizations of the deciles contained in each sub-group. Graph 7-1 graphically depicts the growth of $1.00 invested in each of these capitalization groups.

Size of the Deciles

Table 7-5 reveals that most of the market value of the stocks listed on the NYSE/AMEX/NASDAQ is represented by the top three deciles. Approximately two-thirds of the value is represented by the first decile, which currently consists of 168 stocks. The smallest decile represents just under one percent of the market value of the NYSE/AMEX/NASDAQ. The data in the second column of Table 7-5 are averages across all 84 years. Of course, the proportions represented by the various deciles vary from year to year.

In columns three and four are the number of companies and market capitalization. These present a snapshot of the structure of the deciles near the end of 2009.

The lower portion of Table 7-5 shows the largest firm in each decile and its market capitalization.

Long-Term Returns in Excess of Risk

The Capital Asset Pricing Model (CAPM) does not fully account for the higher returns of small company stocks. Table 7-6 shows the returns in excess of the riskless rate over the past 84 years for each decile of the NYSE/AMEX/NASDAQ.

The CAPM can be expressed as follows:

$$k_s = r_f + (\beta_s \times ERP) \quad (28)$$

where,

k_s = the expected return for company **s**;
r_f = the expected return of the riskless asset;
β_s = the beta of the stock of company **s**; and,
ERP = the expected equity risk premium, or the amount by which investors expect the future return on equities to exceed that on the riskless asset.

Table 7-5: Size-Decile Portfolios of the NYSE/AMEX/NASDAQ
Bounds, Size, and Composition

Decile	Historical Average Percentage of Total Capitalization	Recent Number of Companies	Recent Decile Market Capitalization (in Thousands)	Recent Percentage of Total Capitalization
1-Largest	63.26%	168	$8,067,379,357	63.78%
2	13.94	176	1,681,320,126	13.29
3	7.54	174	802,997,270	6.35
4	4.72	185	566,025,344	4.48
5	3.24	215	435,313,426	3.44
6	2.39	241	319,576,916	2.53
7	1.76	305	281,895,344	2.23
8	1.31	417	197,085,621	1.56
9	1.02	560	178,722,563	1.41
10-Smallest	0.83	1,361	118,046,268	0.93
Mid-Cap 3-5	15.49	574	1,804,335,402	14.27
Low-Cap 6-8	5.45	963	798,557,573	6.31
Micro-Cap 9-10	1.86	1,921	296,768,887	2.35

Data from 1926–2009. Source: Morningstar and CRSP. Calculated (or Derived) based on data from CRSP US Stock Database and CRSP US Indices Database ©2010 Center for Research in Security Prices (CRSP®), The University of Chicago Booth School of Business. Used with permission.

Historical average percentage of total capitalization shows the average, over the last 84 years, of the decile market values as a percentage of the total NYSE/AMEX/NASDAQ calculated each month. Number of companies in deciles, recent market capitalization of deciles and recent percentage of total capitalization are as of September 30, 2009.

Decile	Recent Market Capitalization (in Thousands)	Company Name
1-Largest	$329,725,255	Exxon Mobil Corp
2	14,691,668	Sysco Corp
3	5,936,147	American International Group Inc
4	3,414,634	Resmed Inc
5	2,384,026	Mirant Corp
6	1,600,169	Cypress Semiconductor Corp
7	1,063,308	Enersys
8	684,790	Live Nation Inc
9	431,256	American Reprographics Co
10-Smallest	214,111	Quicksilver Gas Services L P

Source: Morningstar and CRSP. Calculated (or Derived) based on data from CRSP US Stock Database and CRSP US Indices Database ©2010 Center for Research in Security Prices (CRSP®), The University of Chicago Booth School of Business. Used with permission. Market capitalization and name of largest company in each decile as of September 30, 2009.

The amount of an asset's systematic risk is measured by its beta. A beta greater than 1 indicates that the security is riskier than the market, and according to the CAPM equation, investors are compensated for taking on this additional risk. However, based on historical return data on the NYSE/AMEX/NASDAQ decile portfolios, the smaller deciles have had returns that are not fully explainable by the CAPM. This return in excess of CAPM grows larger as one moves from the largest companies in decile 1 to the smallest in decile 10. The excess return is especially pronounced for micro-cap stocks (deciles 9–10). This size related phenomenon has prompted a revision to the CAPM that includes the addition of a size premium.

The CAPM is used here to calculate the CAPM return in excess of the riskless rate and to compare this estimate to historical performance. According to the CAPM, the return on a security should consist of the riskless rate plus an additional return to compensate for the systematic risk of the security. Table 7-6 uses the 84-year arithmetic mean income return component of 20-year government bonds as the historical riskless rate. (However, it is appropriate to match the maturity, or duration, of the riskless asset with the investment horizon.) This CAPM return in excess of the riskless rate is β (beta) multiplied by the realized equity risk premium. The realized equity risk premium is the return that compensates investors for taking on risk equal to the risk of the market as a whole (estimated by the 84-year arithmetic mean return on large company stocks, 11.85 percent, less the historical riskless rate, 5.20 percent). The difference between the excess return predicted by the CAPM and the realized excess return is the size premium, or return in excess of CAPM.

This phenomenon can also be viewed graphically, as depicted in the Graph 7-2. The security market line is based on the pure CAPM without adjusting for the size premium. Based on the risk (or beta) of a security, the expected return should fluctuate along the security market line. However, the expected returns for the smaller deciles of the NYSE/AMEX/NASDAQ lie above the line, indicating that these deciles have had returns in excess of their risk.

For additional information regarding size premia or a more detailed breakdown of the size effect over size-decile portfolios please reference Chapter 7 of the *Ibbotson® SBBI® Valuation Yearbook*.

Table 7-6: Size-Decile Portfolios of the NYSE/AMEX/NASDAQ Long-Term Returns in Excess of CAPM

Decile	Beta*	Arithmetic Mean Return (%)	Actual Return in Excess of Riskless Rate** (%)	CAPM Return in Excess of Riskless Rate† (%)	Size Premium (Return in Excess of CAPM) (%)
Mid-Cap, 3-5	1.12	13.71	8.54	7.45	1.08
Low-Cap, 6-8	1.23	15.20	10.03	8.18	1.85
Micro-Cap, 9-10	1.36	18.23	13.06	9.07	3.99

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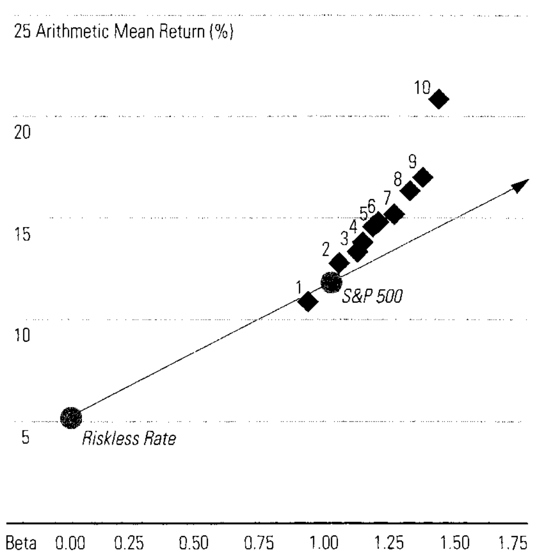
*Betas are estimated from monthly returns in excess of the 30-day U.S. Treasury bill total return, January 1926–December 2009.

**Historical riskless rate measured by the 84-year arithmetic mean income return component of 20-year government bonds (5.18 percent).

†Calculated in the context of the CAPM by multiplying the equity risk premium by beta. The equity risk premium is estimated by the arithmetic mean total return of the S&P 500 (11.85 percent) minus the arithmetic mean income return component of 20-year government bonds (5.18 percent) from 1926–2009.

Graph 7-2: Security Market Line Versus Size-Decile Portfolios of the NYSE/AMEX/NASDAQ



Data from 1926–2009.

Serial Correlation in Small Company Stock Returns

The serial correlation, or first-order autocorrelation, of returns on large capitalization stocks is near zero. [See Table 7-1.] If stock returns are serially correlated, then one can gain some information about future performance based on past returns. For the smallest stocks, the serial correlation is near or above 0.1. This observation bears further examination.

Table 7-7: Size-Decile Portfolios of the NYSE/AMEX/NASDAQ Serial Correlations of Annual Returns in Excess of Decile 1 Returns

Decile	Serial Correlations of Annual Returns in Excess of Decile 1 Return
2	0.25
3	0.21
4	0.20
5	0.25
6	0.33
7	0.26
8	0.30
9	0.27
10	0.36

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To remove the randomizing effect of the market as a whole, the returns for decile 1 are geometrically subtracted from the returns for deciles 2 through 10. The result illustrates that these series differences exhibit greater serial correlation than the decile series themselves. Table 7-7 above presents the serial correlations of the excess returns for deciles 2 through 10. These serial correlations suggest some predictability of smaller company excess returns. However, caution is necessary. The serial correlation of small company excess returns for non-calendar years (February through January, etc.) do not always confirm the results shown here for calendar (January through December) years. The results for the non-calendar years (not shown in this book) suggest that predicting small company excess returns may not be easy.

Table 7-8: Size-Decile Portfolios of the NYSE/AMEX/NASDAQ
Returns in Excess of Decile 1 (%)

First row: Average excess return in percent Second row: Number of times excess return was positive (in 84 years)

Decile	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Total (Jan–Dec)
2	0.86	0.49	-0.03	-0.20	0.09	-0.10	-0.07	0.18	0.02	-0.34	0.10	0.41	1.47
	64	55	39	31	43	41	37	44	44	39	48	46	
3	1.15	0.35	0.05	-0.03	-0.12	-0.13	-0.08	0.37	-0.09	-0.40	0.46	0.34	1.90
	62	53	40	31	37	37	42	50	40	35	47	48	
4	1.35	0.58	-0.06	-0.21	0.11	-0.07	-0.13	0.29	0.09	-0.83	0.31	0.54	2.09
	61	52	40	34	41	41	37	49	43	30	44	48	
5	2.19	0.59	-0.07	-0.14	-0.12	0.02	-0.07	0.31	0.06	-0.82	0.26	0.39	2.73
	62	51	38	36	38	40	43	48	43	33	47	46	
6	2.49	0.43	-0.09	-0.03	0.30	-0.05	-0.17	0.54	0.15	-1.28	0.17	0.31	2.90
	62	52	44	35	40	39	42	49	45	35	44	45	
7	3.09	0.60	-0.10	-0.05	0.15	-0.29	-0.06	0.28	0.26	-1.11	0.11	0.10	2.99
	63	53	43	36	36	34	38	41	46	30	44	41	
8	4.24	0.66	-0.30	-0.30	0.40	-0.39	0.09	0.21	0.06	-1.13	0.10	-0.15	3.84
	62	48	38	33	35	38	40	39	44	34	38	38	
9	5.49	0.85	-0.12	-0.12	0.30	-0.37	0.02	0.17	-0.03	-1.28	-0.03	-0.83	4.32
	63	45	43	33	36	34	37	44	40	33	36	35	
10	8.90	0.89	-0.75	-0.02	0.64	-0.55	0.55	-0.09	0.56	-1.46	-0.58	-1.53	7.41
	77	42	35	35	37	35	40	32	41	29	31	31	

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Seasonality

Unlike the returns on large company stocks, the returns on small company stocks appear to be seasonal. In January, small company stocks often outperform larger stocks by amounts far greater than in any other month.

Table 7-8 shows the returns of capitalization deciles 2 through 10 in excess of the return on decile 1. This table segregates excess returns into months. For each decile and for each month, the exhibit shows both the average excess return as well as the number of times the excess return is positive. These two statistics measure the seasonality of the excess return in different ways. The average excess return illustrates the size of the effect, while the number of positive excess returns shows the reliability of the effect.

Virtually all of the small stock effect occurs in January. The excess outcomes of the other months are on net, mostly negative for small company stocks. Excess returns in January relate to size in a precisely rank-ordered fashion. This "January effect" seems to pervade all size groups.

Endnotes

[1] **Page 85** Rolf W. Banz was the first to document this phenomenon. See Banz, Rolf W., "The Relationship Between Returns and Market Value of Common Stocks," *Journal of Financial Economics*, Volume 9 (1981), pp. 3–18.