

Betas and Their Regression Tendencies

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TABLE I
BETA COEFFICIENTS FOR PORTFOLIOS
OF 100 SECURITIES

Portfolio	Grouping Period	First Subsequent Period
	7/26-6/33	7/33-6/40
1	0.50	0.61
2	0.85	0.96
3	1.15	1.24
4	1.53	1.42
	7/33-6/40	7/40-6/47
1	0.38	0.56
2	0.69	0.77
3	0.90	0.91
4	1.13	1.12
5	1.35	1.31
6	1.68	1.69
	7/40-6/47	7/47-6/54
1	0.43	0.60
2	0.61	0.76
3	0.73	0.88
4	0.86	0.99
5	1.00	1.10
6	1.21	1.21
7	1.61	1.36
	7/47-6/54	7/54-6/61
1	0.36	0.57
2	0.61	0.71
3	0.78	0.88
4	0.91	0.96
5	1.01	1.03
6	1.13	1.13
7	1.26	1.24
8	1.47	1.32
	7/54-6/61	7/61-6/68
1	0.37	0.62
2	0.56	0.68
3	0.72	0.85
4	0.86	0.85
5	0.99	0.95
6	1.11	0.98
7	1.23	1.07
8	1.43	1.25

be expected to be negative may best be explored by analyzing how a security might happen to have one of the 100 lowest estimates of beta. First, if the true beta were in the lowest hundred, the estimated beta would fall in the lowest 100 estimates only if the error in measuring the beta were not too large which roughly translates into more negative than positive errors. Second, if the true beta were not in the lowest 100, the

estimated beta might still be in the lowest 100 estimates if it were measured with a sufficiently large negative error.⁶

Thus, the negative errors in the 100 smallest estimates of beta might be expected to outweigh the positive errors. The same argument except in reverse would apply to the 100 largest estimates. Indeed, it would seem that any portfolio of securities stratified by estimates of beta for which the average of these estimates is not the grand mean of all betas, namely 1.0, would be subject to some order bias. It would also seem that the absolute magnitude of this order bias should be greater, the further the average estimate is from the grand mean. The next section formalizes this intuitive argument and suggests that, if it is not incorrect, it is certainly misleading as to the source of the bias.

III. A FORMAL MODEL

The intuitive explanation of the order bias just given would seem to suggest that the way in which the portfolios are formed caused the bias. This section will argue that the bias is present in the estimated betas for the individual securities and is not induced by the way in which the portfolios are selected. Following this argument will be an analysis of the extent to which this order bias accounts for the observed regression tendency in portfolio betas over time.

A numerical example will serve to illustrate the logic of the subsequent argument and to introduce some required notation.⁷ Assume for the moment that the possible values of beta for an individual security i in period t , β_{it} , are 0.8, 1.0 and 1.2 and that each of these values is equally likely. Assume further that in estimating a beta for an individual security, there is a 0.6 probability that the estimate $\hat{\beta}_{it}$ contains no measurement error, a 0.2 probability that it understates the true β_{it} by 0.2, and a 0.2 probability that it overstates the true value by 0.2. Now in a sample of ten securities whose true betas were all say 0.8, one would expect two estimates of beta to be 0.6, six to be 0.8, and two to be 1.0. These numbers have been transcribed to the first row of Table 2. The second and third rows are similarly constructed by first assuming that the ten securities all had a true value of 1.0 and then of 1.2.

The rows of Table 2 thus correspond to the distribution of the estimated beta, $\hat{\beta}_{it}$, conditional on the true value, β_{it} . It might be noted that the expectation of $\hat{\beta}_{it}$ conditional on β_{it} , $E(\hat{\beta}_{it} | \beta_{it})$, is β_{it} . However, in a sampling situation, an investigator would be faced with an estimate of beta and would want to assess the distribution of the true β_{it} conditional on the estimated $\hat{\beta}_{it}$. Such conditional distributions correspond to the columns of Table 2. It is easily verified that the expectation of β_{it} conditional on $\hat{\beta}_{it}$, $E(\beta_{it} | \hat{\beta}_{it})$ is generally not $\hat{\beta}_{it}$. For example, if $\hat{\beta}_{it}$ were

6. It is theoretically possible that the estimated beta for a security whose true beta does not fall into the lowest 100 to be in the lowest 100 estimates with a positive measurement error if the betas for some of the improperly classified securities are measured with sufficiently large positive errors.

7. The author is indebted to Harry Markowitz for suggesting this numerical example as a way of clarifying the subsequent formal development.

TABLE 2
NUMBER OF SECURITIES CROSS
CLASSIFIED BY β_{it} AND $\hat{\beta}_{it}$

		$\hat{\beta}_{it}$				
		.6	.8	1.0	1.2	1.4
β_{it}	.8	2	6	2		
	1.0		2	6	2	
	1.2			2	6	2

0.8. $E(\beta_{it} | \hat{\beta}_{it} = 0.8)$ would be 0.85 since with this estimate the true beta would be 0.8 with probability 0.75 or 1.0 with probability 0.25.⁸

The estimate $\hat{\beta}_{it}$, therefore, would typically be biased, and it is biased whether or not portfolios are formed. The effect of forming large portfolios is to reduce the random component in the estimate, so that the difference between the estimated portfolio beta and the true portfolio beta can be ascribed almost completely to the magnitude of the bias.

In the spirit of this example, the paper will now develop explicit formulae for the order bias and real non-stationarities over time. Let it be assumed that the betas for individual securities in period t , β_{it} , can be thought of as drawings from a normal distribution with a mean of 1.0 and variance $\sigma^2(\beta_{it})$. The corresponding assumption for the numerical example just discussed would be a trinomial distribution with equal probabilities for each possible value of β_{it} .

Let it additionally be assumed that the estimate, $\hat{\beta}_{it}$, measures β_{it} with error η_{it} , a mean-zero independent normal variate, so that $\hat{\beta}_{it}$ is given by the sum of β_{it} and η_{it} . It immediately follows that β_{it} and $\hat{\beta}_{it}$ are distributed by a bivariate normal distribution. It might be noted that, as formulated, $\sigma^2(\eta_{it})$ need not equal $\sigma^2(\eta_{jt})$, $i \neq j$. Since the empirical work will assume equality, the subsequent theoretical work will also make this assumption even though for the most part it is not necessary. The final assumption is that β_{it} and β_{it+1} are distributed as bivariate normal variates. Because η_{it} is independently distributed, $\hat{\beta}_{it}$ and β_{it+1} will be distributed by a bivariate normal distribution.

That $\hat{\beta}_{it}$ and β_{it+1} are bivariate normal random variables, each with a mean of 1.0, implies the following regression

$$E(\beta_{it+1} | \hat{\beta}_{it}) - 1 = \frac{\text{Cov}(\beta_{it+1}, \hat{\beta}_{it})}{\sigma^2(\hat{\beta}_{it})} (\hat{\beta}_{it} - 1). \quad (1)$$

This regression is similar to the procedure proposed in Blume [3] to adjust the estimated betas for the regression tendency. That procedure was to regress estimates of beta for individual securities from a later period on estimates from an earlier period and to use the coefficients from this regression to adjust future estimates.⁹ The empirical evidence

8. For further and more detailed discussion of the distinction between $E(\beta_{it} | \hat{\beta}_{it})$ and $E(\hat{\beta}_{it} | \beta_{it})$, the reader is referred to Vasicek [7].

9. That the regression of estimated betas from a later period on estimates from an earlier period is similar to (1) follows from noting that $E(\hat{\beta}_{it+1} | \hat{\beta}_{it})$ equals $E(\beta_{it+1} | \hat{\beta}_{it})$ and that $\text{Cov}(\hat{\beta}_{it+1}, \hat{\beta}_{it})$ equals $\text{Cov}(\beta_{it+1}, \hat{\beta}_{it})$. In [3], the grand mean of all betas was estimated in each period and was not assumed equal to 1.0.

presented there indicated that this procedure did improve the accuracy of estimates of future betas, though no claim was made that there might not be better ways to adjust for the regression tendency.

The coefficient of $(\hat{\beta}_n - 1)$ in (1) can be broken down into two components: one of which would correspond to the so-called order bias and the other to a true regression tendency. To achieve this result, note that the covariance of β_{n+1} and $\hat{\beta}_n$ is given by $\text{Cov}(\beta_{n+1}, \beta_n + \eta_{1n})$, which because of the assumed independence of the errors, reduces to the covariance of β_{n+1} and β_n . Making this substitution and replacing $\text{Cov}(\beta_{n+1}, \beta_n)$ by $\rho(\beta_{n+1}, \beta_n)\sigma(\beta_{n+1})\sigma(\beta_n)$, (1) becomes

$$E(\beta_{n+1} | \hat{\beta}_n) - 1 = \frac{\rho(\beta_{n+1}, \beta_n)\sigma(\beta_{n+1})\sigma(\beta_n)}{\sigma^2(\beta_n)} (\hat{\beta}_n - 1). \quad (2)$$

The ratio of $\sigma(\beta_n)\sigma(\beta_{n+1})$ to $\sigma^2(\hat{\beta}_n)$ might be identified with the order bias and the correlation of β_n and β_{n+1} with a true regression.

If the underlying values of beta are stationary over time, the correlation of successive values will be 1.0 and the standard deviations of β_n and β_{n+1} will be the same. Assuming such stationarity and noting then that β_{n+1} equals β_n , equation (2) can be rewritten as¹⁰

$$\begin{aligned} E(\beta_{n+1} | \hat{\beta}_n) - 1 &= E(\beta_n | \hat{\beta}_n) - 1 \\ &= \frac{\sigma^2(\beta_n)}{\sigma^2(\hat{\beta}_n)} (\hat{\beta}_n - 1). \end{aligned} \quad (3)$$

Since $\sigma^2(\beta_n)$ would be less than $\sigma^2(\hat{\beta}_n)$ if beta is measured with any error, the coefficient of $(\hat{\beta}_n - 1)$ would be less than 1.0. This means that the true beta for a security would be expected to be closer to one than the estimated value. In other words, an estimate of beta for an individual security except for an estimate of 1.0 is biased.¹¹

10. Equation (3) can be derived alternatively from the assumption that β_n and $\hat{\beta}_n$ are bivariate normal variables and under the assumption of stationarity β_n will equal β_{n+1} . Vasicek (7) has developed using Bayes' Theorem, an expression for $E(\beta_n | \hat{\beta}_n)$ which can be shown to be mathematically identical to the right hand side of (3). He observed that the procedure used by Merrill Lynch, Pierce, Fenner and Smith, Inc. in their Security Risk Evaluation Service is similar to his expression if $\sigma^2(\eta_{1n})$ is assumed to be the same for all securities. Merrill Lynch's procedure, as he presented it, is to use the coefficient of the cross-sectional regression of $(\hat{\beta}_{n+1} - 1)$ on $(\hat{\beta}_n - 1)$ to adjust future estimates. This adjustment mechanism is in fact the same as (1) or (2) which shows that such a cross sectional regression takes into account real changes in the underlying betas. Only if betas were stationary over time would his formula be similar to Merrill Lynch's.

11. The formula for order bias given by (3) is similar to that which measures the bias in the estimated slope coefficient in a regression on one independent variable measured with error. Explicitly, consider the regression, $y = bx + e$, where e is an independent mean-zero normal disturbance and both y and x are measured in deviate form. Now if x is measured with independent mean-zero error η and y is regressed on $x + \eta$, it is well known that the estimated coefficient, b , will be biased toward zero and the probability limit of b is $\frac{b}{1 + \frac{\sigma^2(\eta)}{\sigma^2(x)}}$. This expression can be

rewritten as $\frac{\sigma^2(x)}{\sigma^2(x + \eta)}$ b . Interpreting x as the true beta less 1.0, the correspondence to (3) is obvious. In this type of regression, one could either adjust the independent variables themselves for bias and thus obtain an unbiased estimate of the regression coefficient or run the regression on the unadjusted variables and then adjust the regression coefficient. The final coefficient will be the same in either case.

In light of this discussion, the paper now reexamines the empirical results of the previous section. The initial task will be to adjust the portfolio betas in the grouping periods for the order bias. After making this adjustment, it will be apparent that much of the regression tendency observed in Table 1 remains. Thus, if (2) is valid, the value of the correlation coefficient is probably not 1.0. The statistical properties of estimates of the portfolio betas in both the grouping and subsequent periods will be examined. The section ends with an additional test that gives further confirmation that much of the regression tendency stems from true non-stationarities in the underlying betas.

To adjust the estimates of beta in the grouping periods for the order bias using (3) would require estimates of the ratio of $\sigma^2(\beta_{1t})$ to $\sigma^2(\hat{\beta}_{1t})$. The sample variance calculated from the estimated betas for all securities in a particular cross-section provides an estimate of $\sigma^2(\hat{\beta}_{1t})$. An estimate of $\sigma^2(\beta_{1t})$ can be derived as the difference between estimates of $\sigma^2(\hat{\beta}_{1t})$ and $\sigma^2(\eta_{1t})$. If the variance of the error in measuring an individual beta is the same for every security, $\sigma^2(\eta_{1t})$ can be estimated as the average over all securities of the squares of the standard error associated with each estimated beta.

In conformity with these procedures, estimates of the ratio of $\sigma^2(\beta_{1t})$ to $\sigma^2(\hat{\beta}_{1t})$ for the five seven-year periods from 1926 through 1961 were respectively 0.92, 0.92, 0.89, 0.82, and 0.75. In other words, an unbiased estimate of the underlying beta for an individual security should be some eight to twenty-five per cent closer to 1.0 than the original estimate. For instance, if $\sigma^2(\beta_{1t})/\sigma^2(\hat{\beta}_{1t})$ were 0.9 and if $\hat{\beta}_{1t}$ were 1.3, an unbiased estimate would be 1.27.

To determine whether the order bias accounted for all of the regression, the estimated betas for the individual securities were adjusted for the order bias using (3) and the appropriate value of the ratio. For the same portfolios of 100 securities examined in the previous section, portfolio betas for the grouping period were recalculated as the average of these adjusted betas. It might be noted that these adjusted portfolio betas could alternatively be obtained by adjusting the unadjusted portfolio betas directly. These adjusted portfolio betas are given in Table 3. For the reader's convenience, the unadjusted portfolio betas and those estimated in the subsequent seven years are reproduced from Table 1.

Before comparing these estimates, let us for the moment consider the statistical properties of the portfolio betas, first in the grouping period and then in the subsequent period. Though unadjusted estimates of the portfolio betas in the grouping period may be biased, they would be expected to be highly "reliable" as that term is used in psychometrics. Thus, regardless of what these estimates measure, they measure it accurately or more precisely their values approximate those which would be expected conditional on the underlying population and how they are calculated. For equally-weighted portfolios, the larger the number of securities, the more reliable would be the estimate.

Specifically, for an equally-weighted portfolio of 100 securities, the standard deviation of the error in the portfolio beta would be one-tenth

As pointed out, standard errors for portfolio betas calculated from those for individual securities assume independence of the errors in estimates. The standard error for a portfolio beta can however be calculated directly without making this assumption of independence by regressing the portfolio returns on the market index. The standard error for the portfolio of the 100 securities with the lowest estimates of beta in the July 1926-June 1933 period was for instance, 0.018, which compares to 0.012 calculated assuming independence. The average standard error of the estimated betas for the four portfolios in this period was also 0.018. The average standard errors of the betas for the portfolios of 100 securities in the four subsequent seven-year periods ending June 1961 were respectively 0.025, 0.027, 0.024, and 0.027. Although these standard errors, not assuming independence, are about 50 per cent larger than before, they are still extremely small compared to the range of possible values for portfolio betas.

For the moment, let us therefore assume that the portfolio betas as estimated in the grouping period before adjustment for order bias are extremely reliable numbers in that whatever they measure, they measure it accurately. In this case, adjusting these portfolio betas for the order bias will give extremely reliable and unbiased estimates of the underlying portfolio beta and therefore these adjusted betas can be taken as very good approximations to the underlying, but unknown, values. The greater the number of securities in the portfolio, the better the approximation will be.

The numerical example in Table 2 gives an intuitive feel for what is happening. Consider a portfolio of a large number of securities whose estimated betas were all 0.8 in a particular sample. It will be recalled that such an estimate requires that the true beta be either 0.8 or 1.0. As the number of securities with estimates of 0.8 increases, one can be more and more confident that 75 per cent of the securities have true betas of 0.8 and 25 per cent have true betas of 1.0 or equivalently that an equally-weighted portfolio of these securities has a beta of 0.85.

The heuristic argument in the prior section might lead some to believe that, contrary to the estimates in the grouping period, there are no order biases associated with the portfolio betas estimated in the subsequent seven years. This belief, however, is not correct. Formally, the portfolios formed in the grouping period are being treated as if they were securities in the subsequent period. To estimate these portfolio betas, portfolio returns were calculated and regressed upon some measure of the market. In this paper so far, these portfolio returns were calculated under an equally-weighted monthly revision strategy in which delisted securities were sold at the last available price and the proceeds reinvested equally in the remaining. Other strategies are, of course, possible.

Since these portfolios are being treated as securities, formula (3) applies, so that there is still some "order bias" present. However, in determining the rate of regression, the appropriate measure of the variance of the errors in the estimates is the variance for the portfolio betas and not for the betas of individual stocks. This fact has the important effect of making the ratio of $\sigma^2(\beta_{it})$ to $\sigma^2(\hat{\beta}_{it})$ much closer to one than for

individual securities. Estimating $\sigma^2(\hat{\beta}_{it})$ and $\sigma^2(\eta_{it})$ for the portfolios formed on the immediately prior period, the value of this ratio for each of the four seven-year periods from 1933 to 1961 was in excess of 0.99 and for the last seven-year period in excess of 0.98. Thus, for most purposes, little error is introduced by assuming that these estimated portfolio betas contain no "order bias" or equivalently that these estimates measure accurately the true portfolio beta.

A comparison of the portfolio betas in the grouping period, even after adjusting for the order bias, to the corresponding betas in the immediately subsequent period discloses a definite regression tendency. This regression tendency is statistically significant at the five per cent level for each of the last three grouping periods, 1940-47, 1947-54, 1954-61.¹² Thus, this evidence strongly suggests that there is a substantial tendency for the underlying values of beta to regress towards the mean over time. Yet, it could be argued that this test is suspect because the formula used in adjusting for the order bias was developed under the assumption that the distributions of beta were normal. This assumption is certainly not strictly correct and it is not clear how sensitive the adjustment is to violations of this assumption.

A more robust way to demonstrate the existence of a true regression tendency is based upon the observation that the portfolio betas estimated in the period immediately subsequent to the grouping period are measured with negligible error and bias. These estimated portfolio betas can be compared to betas for the same portfolios estimated in the second seven years subsequent to the grouping period. These betas, which have been estimated in the second subsequent period and are given in Table 3, disclose again an obvious regression tendency. This tendency is significant at the five per cent level for the last three of the four possible comparisons.¹³

IV. SUMMARY

Beginning with a review of the conventional wisdom, the paper showed that estimated beta coefficients tend to regress towards the grand mean of all betas over time. The next section presented two kinds of empirical analyses which showed that part of this observed regression tendency represented real nonstationarities in the betas of individual securities and that the so-called order bias was not of overwhelming importance.

In other words, companies of extreme risk—either high or low—tend to have less extreme risk characteristics over time. There are two logical

12. This test of significance was based upon the regression $(\hat{\beta}_{it+1} - 1) = b(\hat{\beta}_{it} - 1) + e_{it}$ where $\hat{\beta}_{it}$ has been adjusted for order bias. The estimated coefficients with the t-values measured from 1.0 in parentheses were for the five seven-years chronologically 0.86 (-1.14), 0.94 (-0.88), 0.71 (-3.04), 0.86 (-3.23), and 0.81 (-2.57). Note that even if $\hat{\beta}_{it}$ were measured with substantial independent error contrary to fact, the estimated b would not be biased towards zero because, as footnote 10 shows, the adjustment for the order bias has already corrected for this bias.

13. Using the same regression as in the previous footnote, the estimated coefficient b with the t-value measured from 1.0 in parentheses were for the four possible comparisons in chronological order 0.92 (-0.69), 0.74 (-2.67), 0.62 (-6.86), and 0.98 (-5.51).

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explanations. First, the risk of existing projects may tend to become less extreme over time. This explanation may be plausible for high risk firms, but it would not seem applicable to low risk firms. Second, new projects taken on by firms may tend to have less extreme risk characteristics than existing projects. If this second explanation is correct, it is interesting to speculate on the reasons. For instance, is it a management decision or do limitations on the availability of profitable projects of extreme risk tend to cause the riskiness of firms to regress towards the grand mean over time? Though one could continue to speculate on the forces underlying this tendency of risk—as measured by beta coefficients—to regress towards the grand mean over time, it remains for future research to determine the explicit reasons.

REFERENCES

1. Fischer Black, Michael C. Jensen and Myron Scholes. "The Capital Asset Pricing Model: Some Empirical Tests," in Michael C. Jensen, ed., *Studies in the Theory of Capital Markets*. New York: Praeger Publishing, 1972.
2. Marshall Blume. "Portfolio Theory: A Step Towards Its Practical Application," *Journal of Business* (April 1970).
3. ———. "On the Assessment of Risk," *Journal of Finance* (March 1971).
4. ——— and Irwin Friend. "A New Look at the Capital Asset Pricing Model," *Journal of Finance* (March 1973).
5. Eugene F. Fama and James D. MacBeth. "Risk, Return and Equilibrium: Empirical Tests," *Journal of Political Economy* (May 1973).
6. Lawrence Fisher. "Some New Stock-Market Indexes," *Journal of Business* (January 1966), supplement.
7. Oldrich A. Vasicek. "A Note on Using Cross-Sectional Information in Bayesian Estimation of Security Betas," *Journal of Finance* (December 1973).

Pauline Ahern

From: Frank Hanley <fhanley@ausinc.com>
To: Pauline Ahern <pahern@ausinc.com>
Sent: Thursday, August 31, 2000 4:20 PM
Subject: Fw: ECAPM

— Original Message —

From: Frank Hanley
To: profmorin@msn.com
Sent: Thursday, August 31, 2000 4:18 PM
Subject: ECAPM

Dr. Morin,

Quite some time ago I sent you e mail about the ECAPM. You replied that critics were wrong when they say that using the ECAPM with adjusted beta is a double counting. You said that you would provide me with some proof. Could you please send me something or point me to specific empirical support that use of adjusted beta in the ECAPM is not double counting ?

I know that you are a very busy man so I give you many thanks in advance for any time you take in responding to me.

Appreciatively,

Frank Hanley

Pauline Ahern

From: Frank Hanley <fhanley@ausinc.com>
To: Pauline Ahern <pahern@ausinc.com>
Sent: Friday, September 01, 2000 1:19 PM
Attach: response to F.Hanley.doc
Subject: Fw: ECAPM

----- Original Message -----

From: profmorin
To: fhanley@ausinc.com
Sent: Friday, September 01, 2000 12:51 PM
Subject: Re: ECAPM

Dear Frank:

I have attached a response to your concern. I also point out that the New York PSC has endorsed the Morin ECAPM following the massive generic cost of capital hearing of a few years ago. I have the exact cite if you need it.

----- Original Message -----

From: Frank Hanley
To: profmorin@msn.com
Sent: Thursday, August 31, 2000 4:18 PM
Subject: ECAPM

Dr. Morin,

Quite some time ago I sent you e mail about the ECAPM. You replied that critics were wrong when they say that using the ECAPM with adjusted beta is a double counting. You said that you would provide me with some proof. Could you please send me something or point me to specific empirical support that use of adjusted beta in the ECAPM is not double counting ?

I know that you are a very busy man so I give you many thanks in advance for any time you take in responding to me.

Appreciatively,

Frank Hanley

MORIN ECAPM

Some have argued that the Morin ECAPM constitutes a double beta adjustment. I do not share the view that the ECAPM is equivalent to a beta adjustment.

There are two distinct separate issues involved when implementing the CAPM. First, given the validity of the standard CAPM, what is the best proxy for expected beta? Second, and more fundamentally, does the standard form of the CAPM provide the best explanation of the risk-return relationship observed on capital markets?

i. Beta measurement

Unadjusted raw betas are inappropriate to use in a CAPM analysis. The raw unadjusted beta is not the appropriate measure of market risk to use. Current stock prices reflect expected risk, that is, expected beta, rather than historical risk or historical beta. Historical betas, whether raw or adjusted, are only surrogates for expected beta. The best of the two surrogates is adjusted beta a la Value Line, Merrill Lynch, and Bloomberg betas.

ii. Standard CAPM

There have been countless empirical tests of the CAPM to determine to what extent security returns and betas are related in the manner predicted by the CAPM. The results of the tests support the idea that beta is related to security returns, that the risk-return tradeoff is positive, and that the relationship is linear. The contradictory finding is that the risk-return tradeoff is not as steeply sloped as the predicted CAPM. That is, low-beta securities earn returns somewhat higher than the CAPM would predict, and high-beta securities earn less than predicted. This is one of the most well-known results in finance. A CAPM-based estimate of cost of capital underestimates the return required from low-beta securities and overstates the return from high-beta securities, based on the empirical evidence. The empirical form of the CAPM refines the standard form of the CAPM to account for this phenomenon.

Thus, I do not share the view that the ECAPM is equivalent to a beta adjustment. For utility stocks with betas less than one, the CAPM understates the return. The ECAPM allows for the CAPM's inherent bias by ascribing a higher intercept and flatter slope to the CAPM. The ECAPM is a return (Y-axis, vertical axis) adjustment. It is not a beta risk (x-axis, horizontal) adjustment. The ECAPM is not an attempt to increase the beta estimate, which would be a horizontal x-axis adjustment. The ECAPM is a return adjustment rather than a risk adjustment.

There is a huge financial literature which supports both the use of the ECAPM and the use of adjusted betas. The empirical support for adjusted betas and for the ECAPM is summarized in Chapter 13 of my book, Regulatory Finance, Public Utility Reports Inc., Arlington, Va., 1994.

With few exceptions, the empirical studies support the finding that the implied intercept term exceeds the risk-free rate and the slope term is less than predicted by the CAPM.

Consumers Illinois Water Company
Market Capitalization of Consumers Illinois Water Company,
Mr. McNally's Water Utility Sample and
Mr. McNally's Comparable Sample

1	2	3	4	5	6	7		
Company	Stock Exchange Listing	Common Stock Shares Outstanding (1) (millions)	Book Value per Share (1)	Total Common Equity (4) (millions)	Closing Stock Market Price on August 9, 2000 (1)	Market-to-Book Ratio at August 9, 2000 (5)	Market Capitalization on August 9, 2000 (6) (millions)	1999 Total Capitalization (incl. Short-Term Debt) (millions)
Consumers Illinois Water Company	NA	NA (2)	NA (2)	\$41,854 (3)	NA	186.8 % (5)	\$78,183 (7)	\$82,145
Mr. McNally's Water Utility Sample								
American States Water Company	NYSE	8,958	\$17.73	\$158,846	\$27,000	152.3	\$241,868	\$349,469
American Water Works Inc	NYSE	97,194	\$18.82	\$1,834,798	\$25,000	148.6	\$2,429,850	\$4,399,925
Artesian Resources	NDQ	1,998	\$16.06	\$32,084	\$23,250	144.8	\$46,454	\$76,719
Connecticut Water Service Inc.	NDQ	4,839	\$12.91	\$62,495	\$32,000	247.8	\$154,848	\$131,271
Middlesex Water Company	NDQ	5,001	\$14.09	\$70,489	\$27,750	198.9	\$138,778	\$158,084
Pennichuck Corporation	NDQ	1,747	\$15.03	\$26,257	\$24,000	159.7	\$41,928	\$54,818
Philadelphia Suburban Corporation	NYSE	41,013	\$8.89	\$364,537	\$22,875	257.4	\$938,172	\$897,916
Average		23,000	\$14.51	\$336.00	\$25,982	186.8 %	\$570,271	\$667,029
Mr. McNally's Comparable Sample								
Connecticut Water Service Inc.	NDQ	4,839	\$12.91	\$62,495	\$32,000	247.8	\$154,848	\$131,271
Constellation Energy Corporation	NYSE	149,556	\$20.01	\$2,993,000	\$36,625	183.0	\$5,477,489	\$6,938,200
Hawaiian Electric Industries	NYSE	32,213	\$28.31	\$847,586	\$31,938	121.4	\$1,028,819	\$4,062,491
Idacorp Inc.	NYSE	37,612	\$20.02	\$752,970	\$23,688	188.3	\$1,417,521	\$1,789,197
Kansas City Power & Light	NYSE	61,909	\$13.97	\$864,644	\$26,063	186.6	\$1,613,534	\$2,107,147
Northwest Natural Gas Company	NYSE	25,092	\$17.12	\$429,596	\$23,563	137.6	\$581,243	\$965,688
Pennichuck Corporation	NDQ	1,747	\$15.03	\$26,257	\$24,000	159.7	\$41,928	\$54,818
Philadelphia Suburban Corporation	NYSE	41,013	\$8.89	\$364,537	\$22,875	257.4	\$938,172	\$897,916
Potomac Electric Power	NYSE	118.5	\$18.12	\$1,910,300	\$25,938	160.9	\$3,073,653	\$5,370,100
Public Service Enterprise	NYSE	216,416	\$18.46	\$3,996,000	\$95,781	193.8	\$7,743,662	\$12,824,000
RGS Energy Group Inc.	NYSE	35,943	\$21.43	\$770,202	\$24,375	113.6	\$878,111	\$1,705,810
Average		68,000	\$17.30	\$1,183.00	\$28,166	177.3 %	\$2,086,997	\$3,349,694

- Notes: (1) Column 3 / Column 1.
(2) From InterQuote, Inc.
(3) Column 4 / Column 1.
(4) Column 1 * Column 4.
(5) Not Available
(6) The market-to-book ratio of Consumers Illinois Water Company at August 9, 2000 is assumed to be equal to the average market-to-book ratio at August 9, 2000 of Mr. McNally's water utility sample.
(7) Assuming that Consumer Illinois Water company's common stock, if traded, would trade at a market-to-book ratio equal to the average market-to-book ratio at August 9, 2000 of Mr. McNally's water utility sample, 186.8%, CIWC's consolidated market capitalization at August 9, 2000 would have been \$82,145

Source of Information: Standard & Poor's Compustat Services, Inc., PC Plus Data Base.
From Schedule D-1, page 1 of the Company's filing

STOCKS, BONDS, BILLS AND INFLATION:

2000 YEARBOOK

SBBI

VALUATION EDITION

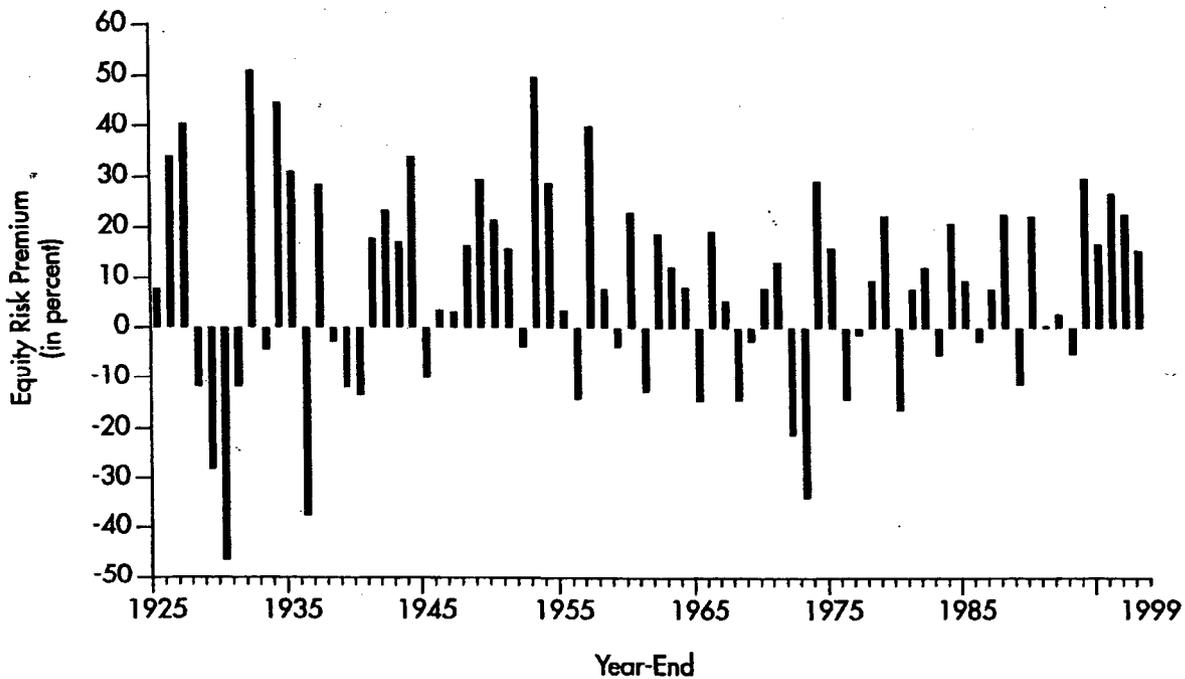
changes in bonds due to unanticipated changes in yields introduce price risk into the total return. Therefore, the total return on the bond series does not represent the riskless rate of return. There is no evidence that investors expect the historical trend of bond capital losses to be repeated in the future (otherwise, bond prices would be adjusted accordingly). Therefore, historical total returns are biased downward as indicators of future expectations. The income return better represents the unbiased estimate of the purely riskless rate of return, since an investor can hold a bond to maturity and be entitled to the income return with no capital loss.

Arithmetic versus Geometric Means

The equity risk premium data presented in this book are arithmetic average risk premia as opposed to geometric average risk premia. The arithmetic average equity risk premium can be demonstrated to be most appropriate when discounting future cash flows. For use as the expected equity risk premium in either the CAPM or the building block approach, the arithmetic mean or the simple difference of the arithmetic means of stock market returns and riskless rates is the relevant number. This is because both the CAPM and the building block approach are additive models, in which the cost of capital is the sum of its parts. The geometric average is more appropriate for reporting past performance, since it represents the compound average return.

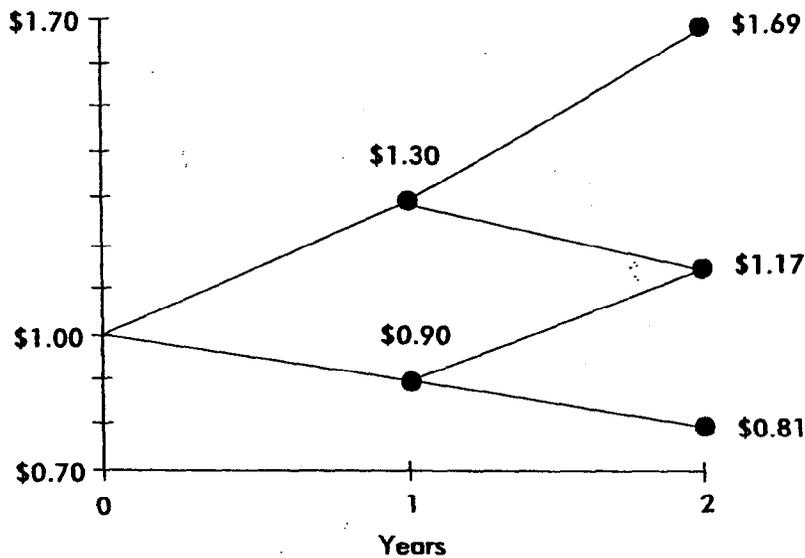
The argument for using the arithmetic average is quite straightforward. In looking at projected cash flows, the equity risk premium that should be employed is the equity risk premium that is expected to actually be incurred over the future time periods. Graph 3-3 shows the realized equity risk premium for each year based on the returns of the S&P 500 and the income return on long-term government bonds. (The actual, observed difference between the return on the stock market and the riskless rate is known as the *realized equity risk premium*.) There is considerable volatility in the year-by-year statistics. At times the realized equity risk premium is even negative.

Graph 3-3: Realized Equity Risk Premium Per Year (1926-1999)



To illustrate how the arithmetic mean is more appropriate than the geometric mean in discounting cash flows, suppose the expected return on a stock is 10 percent per year with a standard deviation of 20 percent. Also assume that only two outcomes are possible each year— +30 percent and -10 percent (*i.e.*, the mean plus or minus one standard deviation). The probability of occurrence for each outcome is equal. The growth of wealth over a two-year period is illustrated in Graph 3-4.

Graph 3-4: Growth of Wealth Example



The most common outcome of \$1.17 is given by the geometric mean of 8.2 percent. Compounding the possible outcomes as follows derives the geometric mean:

$$[(1+0.30) \times (1-0.10)]^{\frac{1}{2}} - 1 = 0.082$$

However, the expected value is predicted by compounding the arithmetic, not the geometric, mean. To illustrate this, we need to look at the probability-weighted average of all possible outcomes:

(0.25 × \$1.69) =	\$0.4225
+ (0.50 × \$1.17) =	\$0.5850
+ (0.25 × \$0.81) =	\$0.2025
Total	\$1.2100

Therefore, \$1.21 is the probability-weighted expected value. The rate that must be compounded to achieve the terminal value of \$1.21 after 2 years is 10 percent, the arithmetic mean:

$$\$1 \times (1+0.10)^2 = \$1.21$$

The geometric mean, when compounded, results in the median of the distribution:

$$\$1 \times (1+0.082)^2 = \$1.17$$

The arithmetic mean equates the expected future value with the present value; it is therefore the appropriate discount rate.

Appropriate Historical Time Period

The equity risk premium can be estimated using any historical time period. For the U.S., market data exists at least as far back as the late 1800s. Therefore, it is possible to estimate the equity risk premium using data that covers roughly the past 100 years.

The Ibbotson Associates equity risk premium covers the time period from 1926 to the present. The original data source for the time series comprising the equity risk premium is the Center for Research in Security Prices. CRSP chose to begin their analysis of market returns with 1926 for two main reasons. CRSP determined that the time period around 1926 was approximately when quality financial data became available. They also made a conscious effort to include the period of extreme market volatility from the late twenties and early thirties; 1926 was chosen because it includes one full business cycle of data before the market crash of 1929. These are the most basic reasons why Ibbotson Associates' equity risk premium calculation window starts in 1926.

Implicit in using history to forecast the future is the assumption that investors' expectations for future outcomes conform to past results. This method assumes that the price of taking on risk changes only slowly, if at all, over time. This "future equals the past" assumption is most applicable to a random time-series variable. A time-series variable is random if its value in one period is independent of its value in other periods.

Does the Equity Risk Premium Revert to Its Mean over Time?

Some have argued that the estimate of the equity risk premium is upwardly biased since the stock market is currently priced high. In other words, since there have been several years with extraordinarily high market returns and realized equity risk premia, the expectation is that returns and realized equity risk premia will be lower in the future, bringing the average back to a normalized level. This argument relies on several studies that have tried to determine whether reversion to the mean exists in stock market prices and the equity risk premium.² Several academics contradict each other on this topic; moreover, the evidence supporting this argument is neither conclusive nor compelling enough to make such a strong assumption.

² Fama, Eugene F., and Kenneth R. French. "Permanent and Temporary Components of Stock Prices," *Journal of Political Economy*, April 1988, pp. 246-273. Poterba, James M., and Lawrence H. Summers. "Mean Reversion in Stock Prices," *Journal of Financial Economics*, October 1988, pp. 27-59. Lo, Andrew W., and A. Craig MacKinlay. "Stock Market Prices Do Not Follow Random Walks: Evidence from a Simple Specification Test," *The Review of Financial Studies*, Spring 1988, pp. 41-66. Finnerty, John D., and Dean Leistikow. "The Behavior of Equity and Debt Risk Premiums: Are They Mean Reverting and Downward-Trending?" *The Journal of Portfolio Management*, Summer 1993, pp. 73-84. Ibbotson, Roger G., and Scott L. Lummer. "The Behavior of Equity and Debt Risk Premiums: Comment," *The Journal of Portfolio Management*, Summer 1994, pp. 98-100. Finnerty, John D., and Dean Leistikow. "The Behavior of Equity and Debt Risk Premiums: Reply to Comment," *The Journal of Portfolio Management*, Summer 1994, pp. 101-102.

Our own empirical evidence suggests that the yearly difference between the stock market total return and the U.S. Treasury bond income return in any particular year is random. Graph 3-3, presented earlier, illustrates the randomness of the realized equity risk premium.

A statistical measure of the randomness of a return series is its serial correlation. Serial correlation (or autocorrelation) is defined as the degree to which the return of a given series is related from period to period. A serial correlation near positive one indicates that returns are predictable from one period to the next period and are positively related. That is, the returns of one period are a good predictor of the returns in the next period. Conversely, a serial correlation near negative one indicates that the returns in one period are inversely related to those of the next period. A serial correlation near zero indicates that the returns are random or unpredictable from one period to the next. Table 3-3 contains the serial correlation of the market total returns, the realized long-horizon equity risk premium, and inflation.

Table 3-3: Interpretation of Annual Serial Correlations (1926–1999)

<u>Series</u>	<u>Serial Correlation</u>	<u>Interpretation</u>
Large Company Stock Total Returns	0.01	Random
Equity Risk Premium	0.02	Random
Inflation Rates	0.65	Trend

The significance of this evidence is that the realized equity risk premium next year will not be dependent on the realized equity risk premium from this year. That is, there is no discernable pattern in the realized equity risk premium—it is virtually impossible to forecast next year's realized risk premium based on the premium of the previous year. For example, if this year's difference between the riskless rate and the return on the stock market is higher than last year's, that does not imply that next year's will be higher than this year's. It is as likely to be higher as it is lower. The best estimate of the expected value of a variable that has behaved randomly in the past is the average (or arithmetic mean) of its past values.

Table 3-4 also indicates that the equity risk premium varies considerably by decade, from a high of 17.9 percent in the 1950s to a low of 0.3 percent in the 1970s. This look at the historical equity risk premium reveals no observable pattern.

Table 3-4: Long-Horizon Equity Risk Premium by Decade (1926-1999)

1920s*	1930s	1940s	1950s	1960s	1970s	1980s	1990s
17.6%	2.3%	8.0%	17.9%	4.2%	0.3%	7.9%	12.1%

* Based on the period 1926-1929.

Finnerty and Leistikow perform more econometrically sophisticated tests of mean reversion in the equity risk premium. Their tests demonstrate that—as we suspected from our simpler tests—the equity risk premium that was realized over 1926 to the present was almost perfectly free of mean reversion and had no statistically identifiable time trends.³ Lo and MacKinlay conclude, “the rejection of the random walk for weekly returns does not support a mean-reverting model of asset prices.”

Choosing an Appropriate Historical Period

The estimate of the equity risk premium depends on the length of the data series studied. A proper estimate of the equity risk premium requires a data series long enough to give a reliable average without being unduly influenced by very good and very poor short-term returns. When calculated using a long data series, the historical equity risk premium is relatively stable.⁴ Furthermore, because an average of the realized equity risk premium is quite volatile when calculated using a short history, using a long series makes it less likely that the analyst can justify any number he or she wants. The magnitude of how shorter periods can affect the result will be explored later in this chapter.

³ Though the study performed by Finnerty and Leistikow demonstrates that the traditional equity risk premium exhibits no mean reversion or drift, they conclude that, “the processes generating these risk premiums are generally mean-reverting.” This conclusion is completely unrelated to their statistical findings and has received some criticism. In addition to examining the traditional equity risk premia, Finnerty and Leistikow include analyses on “real” risk premia as well as separate risk premia for income and capital gains. In their comments on the study, Ibbotson and Lummer show that these “real” risk premia adjust for inflation twice, “creating variables with no economic content.” In addition, separating income and capital gains does not shed light on the behavior of the risk premia as a whole.

⁴ This assertion is further corroborated by data presented in *Global Investing: The Professional's Guide to the World of Capital Markets* (by Roger G. Ibbotson and Gary P. Brinson and published by McGraw-Hill, New York). Ibbotson and Brinson constructed a stock market total return series back to 1790. Even with some uncertainty about the accuracy of the data before the mid-nineteenth century, the results are remarkable. The real (adjusted for inflation) returns that investors received during the three 50-year periods and one 51-year period between 1790 and 1990 did not differ greatly from one another (that is, in a statistically significant amount). Nor did the real returns differ greatly from the overall 201-year average. This finding implies that because real stock-market returns have been reasonably consistent over time, investors can use these past returns as reasonable bases for forming their expectations of future returns.

Some analysts estimate the expected equity risk premium using a shorter, more recent time period on the basis that recent events are more likely to be repeated in the near future; furthermore, they believe that the 1920s, 1930s, and 1940s contain too many unusual events. This view is suspect because all periods contain "unusual" events. Some of the most unusual events of this century took place quite recently, including the inflation of the late 1970s and early 1980s, the October 1987 stock market crash, the collapse of the high-yield bond market, the major contraction and consolidation of the thrift industry, the collapse of the Soviet Union, and the development of the European Economic Community—all of these happened in the last 20 years.

It is even difficult for economists to predict the economic environment of the future. For example, if one were analyzing the stock market in 1987 before the crash, it would be statistically improbable to predict the impending short-term volatility without considering the stock market crash and market volatility of the 1929–1931 period.

Without an appreciation of the 1920s and 1930s, no one would believe that such events could happen. The 74-year period starting with 1926 is representative of what *can* happen: it includes high and low returns, volatile and quiet markets, war and peace, inflation and deflation, and prosperity and depression. Restricting attention to a shorter historical period underestimates the amount of change that could occur in a long future period. Finally, because historical event-types (not specific events) tend to repeat themselves, long-run capital market return studies can reveal a great deal about the future. Investors probably expect "unusual" events to occur from time to time, and their return expectations reflect this.

A Look at the Historical Results

It is interesting to take a look at the realized returns and realized equity risk premium in the context of the above discussion. Table 3-5 shows the average stock market return and the average (arithmetic mean) realized long-horizon equity risk premium over various historical time periods. Similarly, Graph 3-5 shows the average (arithmetic mean) realized equity risk premium calculated through 1999 for different starting dates. The table and the graph both show that using a longer historical period provides a more stable estimate of the equity risk premium. The reason is that any unique period will not be weighted heavily in an average covering a longer historical period. It better represents the probability of these unique events occurring over a long period of time.

Table 3-5: Stock Market Return and Equity Risk Premium Over Time (1926-1999)

Period Length	Period Dates	Large Company Stock Arithmetic Mean Total Return	Long-Horizon Equity Risk Premium
74 years	1926-1999	13.3%	8.1%
70 years	1930-1999	12.8%	7.5%
60 years	1940-1999	14.1%	8.4%
50 years	1950-1999	14.8%	8.5%
40 years	1960-1999	13.3%	6.1%
30 years	1970-1999	14.9%	6.7%
20 years	1980-1999	18.6%	10.0%
15 years	1985-1999	19.6%	11.9%
10 years	1990-1999	19.0%	12.1%
5 years	1995-1999	28.7%	22.3%

Looking carefully at Graph 3-5 will clarify this point. The graph shows the realized equity risk premium for a series of time periods through 1999, starting with 1926. In other words, the first value on the graph represents the average realized equity risk premium over the period 1926-1999. The next value on the graph represents the average realized equity risk premium over the period 1927-1999, and so on, with the last value representing the average over the most recent five years, 1995-1999. Concentrating on the left side of Graph 3-5, one notices that the realized equity risk premium, when measured over long periods of time, is relatively stable. In viewing the graph from left to right, moving from longer to shorter historical periods, one sees that the value of the realized equity risk premium begins to decline significantly. Why does this occur? The reason is that the severe bear market of 1973-1974 is receiving proportionately more weight in the shorter, more recent average. If you continue to follow the line to the right, however, you will also notice that when 1973 and 1974 fall out of the recent average, the realized equity risk premium jumps up by nearly three percent.