SELF-TEST QUESTIONS

Differentiate between a closely held corporation and a publicly owned corporation.
Differentiate between a listed stock and an unlisted stock.
Differentiate between primary and secondary markets.
What is an IPO?

COMMON STOCK VALUATION

Common stock represents an ownership interest in a corporation, but to the typical investor, a share of common stock is simply a piece of paper characterized by two features:

1. It entitles its owner to dividends, but only if the company has earnings out of which dividends can be paid, and only if management chooses to pay dividends rather than retaining and reinvesting all the earnings. Whereas a bond contains a *promise* to pay interest, common stock provides no such promise—if you own a stock, you may *expect* a dividend, but your expectations may not in fact be met. To illustrate, Long Island Lighting Company (LILCO) had paid dividends on its common stock for more than 50 years, and people expected those dividends to continue. However, when the company encountered severe problems a few years ago, it stopped paying dividends. Note, though, that LILCO continued to pay interest on its bonds; if it had not, then it would have been declared bankrupt, and the bondholders could potentially have taken over the company.
2. Stock can be sold at some future date, hopefully at a price greater than the purchase price. If the stock is actually sold at a price above its purchase price, the investor will receive a *capital gain*. Generally, at the time people buy common stocks, they do expect to receive capital gains; otherwise, they would not buy the stocks. However, after the fact, one can end up with capital losses rather than capital gains. LILCO's stock price dropped from $17.50 to $3.75 in one year, so the *expected* capital gain on that stock turned out to be a huge *actual* capital loss.

Definitions of Terms Used in Stock Valuation Models

Common stocks provide an expected future cash flow stream, and a stock's value is found in the same manner as the values of other financial assets—namely, as the present value of the expected future cash flow stream. The expected cash flows consist of two elements: (1) the dividends expected in each year and (2) the price investors expect to receive when they sell the stock. The expected final stock price includes the return of the original investment plus an expected capital gain.

We saw in Chapter 1 that managers seek to maximize the values of their firms' stocks. A manager's actions affect both the stream of income to investors and the riskiness of that stream. Therefore, managers need to know how alternative actions are likely to affect stock prices. At this point we develop some models to help show how the value of a share of stock is determined. We begin by defining the following terms:

D_t = dividend the stockholder *expects* to receive at the end of Year t. D_0 is the most recent dividend, which has already been paid; D_1 is the first dividend expected, and it will be paid at the end of this year; D_2 is the dividend expected at the end of two years; and so forth. D_1 represents the first cash flow a new purchaser of the stock will receive. Note that D_0, the dividend which has just been paid, is known with certainty. However, all future div

dends are expected values, so the estimate of D_t may differ among investors.[5]

P_0 = actual **market price** of the stock today.

$\hat{P}_t$ = expected price of the stock at the end of each Year t (pronounced "P hat t"). $\hat{P}_0$ is the **intrinsic,** or *theoretical,* **value** of the stock today as seen by the particular investor doing the analysis; $\hat{P}_1$ is the price expected at the end of one year; and so on. Note that $\hat{P}_0$ is the intrinsic value of the stock today based on a particular investor's estimate of the stock's expected dividend stream and the riskiness of that stream. Hence, whereas the market price P_0 is fixed and is identical for all investors, $\hat{P}_0$ could differ among investors depending on how optimistic they are regarding the company. The caret, or "hat," is used to indicate that $\hat{P}_t$ is an estimated value. $\hat{P}_0$, the individual investor's estimate of the intrinsic value today, could be above or below P_0, the current stock price, but an investor would buy the stock only if his or her estimate of $\hat{P}_0$ were equal to or greater than P_0.

Since there are many investors in the market, there can be many values for $\hat{P}_0$. However, we can think of a group of "average," or "marginal," investors whose actions actually determine the market price. For these marginal investors, P_0 must equal $\hat{P}_0$; otherwise, a disequilibrium would exist, and buying and selling in the market would change P_0 until $P_0 = \hat{P}_0$ for a marginal investor.

g = expected **growth rate** in dividends as predicted by a marginal investor. If dividends are expected to grow at a constant rate, g is also equal to the expected rate of growth in earnings and in the stock's price. Different investors may use different g's to evaluate a firm's stock, but the market price, P_0, is set on the basis of the g estimated by marginal investors.

k_s = minimum acceptable, or **required rate of return,** on the stock, considering both its riskiness and the returns available on other investments. Again, this term generally relates to marginal investors. The determinants of k_s include the real rate of return, expected inflation, and risk premiums, as discussed in Chapter 5.

$\hat{k}_s$ = **expected rate of return** which an investor who buys the stock expects to receive. $\hat{k}_s$ (pronounced "k hat s") could be above or below k_s, but one would buy the stock only if $\hat{k}_s$ were equal to or greater than k_s.

$\bar{k}_s$ = **actual,** or **realized,** *after-the-fact* **rate of return,** pronounced "k bar s." You may *expect* to obtain a return of $\hat{k}_s$ = 15 percent if you buy Exxon stock today, but if the market goes down, you may end up next year with an actual realized return that is much lower, perhaps even negative.

D_1/P_0 = expected **dividend yield** on the stock during the coming year. If the stock is expected to pay a dividend of D_1 = \$1 during the next 12 months, and if its current price is P_0 = \$10, then the expected dividend yield is \$1/\$10 = 0.10 = 10%.

[5]Stocks generally pay dividends quarterly, so theoretically we should evaluate them on a quarterly basis. However, in stock valuation, most analysts work on an annual basis because the data generally are not precise enough to warrant refinement to a quarterly model. For additional information on the quarterly model, see Charles M. Linke and J. Kenton Zumwalt, "Estimation Biases in Discounted Cash Flow Analysis of Equity Capital Cost in Rate Regulation," *Financial Management,* Autumn 1984, 15–21.

$\frac{\hat{P}_1 - P_0}{P_0}$ = expected **capital gains yield** on the stock during the coming year. If the stock sells for \$10 today, and if it is expected to rise to \$10.50 at the end of one year, then the expected capital gain is $\hat{P}_1 - P_0 = \$10.50 - \$10.00 = \$0.50$, and the expected capital gains yield is $\$0.50/\$10 = 0.05 = 5\%$.

Expected total return = $\hat{k}_s$ = expected dividend yield (D_1/P_0) plus expected capital gains yield $[(\hat{P}_1 - P_0)/P_0]$. In our example, the **expected total return** = $\hat{k}_s = 10\% + 5\% = 15\%$.

Expected Dividends as the Basis for Stock Values

In our discussion of bonds in Chapter 8, we found the value of a bond as the present value of interest payments over the life of the bond plus the present value of the bond's maturity (or par) value:

$$V_B = \frac{INT}{(1+k_d)^1} + \frac{INT}{(1+k_d)^2} + \cdots + \frac{INT}{(1+k_d)^N} + \frac{M}{(1+k_d)^N}.$$

Stock prices are likewise determined as the present value of a stream of cash flows, and the basic stock valuation equation is similar to the bond valuation equation. What are the cash flows that corporations provide to their stockholders? First, think of yourself as an investor who buys a stock with the intention of holding it (in your family) forever. In this case, all that you (and your heirs) will receive is a stream of dividends, and the value of the stock today is calculated as the present value of an infinite stream of dividends:

$$\begin{aligned}\text{Value of stock} = \hat{P}_0 &= \text{PV of expected future dividends}\\ &= \frac{D_1}{(1+k_s)^1} + \frac{D_2}{(1+k_s)^2} + \cdots + \frac{D_\infty}{(1+k_s)^\infty}\\ &= \sum_{t=1}^{N} \frac{D_t}{(1+k_s)^t}. \qquad (9\text{-}1)\end{aligned}$$

What about the more typical case, where you expect to hold the stock for a finite period and then sell it—what will be the value of $\hat{P}_0$ in this case? Unless the company is likely to be liquidated and thus to disappear, *the value of the stock is again determined by Equation 9-1.* To see this, recognize that for any individual investor, the expected cash flows consist of expected dividends plus the expected sale price of the stock. However, the sale price the current investor receives will depend on the dividends some future investor expects. Therefore, for all present and future investors in total, expected cash flows must be based on expected future dividends. Put another way, unless a firm is liquidated or sold to another concern, the cash flows it provides to its stockholders will consist only of a stream of dividends. Therefore, the value of a share of its stock must be established as the present value of that expected dividend stream.

The general validity of Equation 9-1 can also be confirmed by asking the following question: Suppose I buy a stock and expect to hold it for one year. I will receive dividends during the year plus the value $\hat{P}_1$ when I sell out at the end of the year. But what will determine the value of $\hat{P}_1$? The answer is that it will be determined as the present value of the dividends expected during Year 2 plus the stock price at the end of that year, which, in turn, will be determined as the present value of another set of future

dividends and an even more distant stock price. This process can be continued ad infinitum, and the ultimate result is Equation 9-1.[6]

Equation 9-1 is a generalized stock valuation model in the sense that the time pattern of D_t can be anything: D_t can be rising, falling, or constant, it can be fluctuating randomly, or it can even be zero for several years, and Equation 9-1 will still hold. Often, however, the projected stream of dividends is expected to follow a systematic pattern, in which case we can develop a simplified (that is, easier to evaluate) version of the stock valuation model expressed in Equation 9-1. In the following sections, we consider the cases of zero growth, constant growth, and nonconstant growth.

Stock Values with Zero Growth

Suppose dividends are not expected to grow at all but to remain constant. Here we have a **zero growth stock,** for which the dividends expected in future years are equal to some constant amount—that is, $D_1 = D_2 = D_3$ and so on. Therefore, we can drop the subscripts on D and rewrite Equation 9-1 as follows:

$$\hat{P}_0 = \frac{D}{(1+k_s)^1} + \frac{D}{(1+k_s)^2} + \cdots + \frac{D}{(1+k_s)^\infty}. \tag{9-1a}$$

As we noted in Chapter 7 in connection with the British consol bond, a security that is expected to pay a constant amount each year forever is called a perpetuity. *Therefore, a zero growth stock is a perpetuity.*

Although a zero growth stock is expected to provide a constant stream of dividends into the indefinite future, each dividend has a smaller present value than the preceding one, and as the years get very large, the present value of the future dividends approaches zero. To illustrate, suppose D = \$1.15 and k_s = 13.4%. We can rewrite Equation 9-1a as follows:

$$\begin{aligned}\hat{P}_0 &= \frac{\$1.15}{(1.134)^1} + \frac{\$1.15}{(1.134)^2} + \frac{\$1.15}{(1.134)^3} + \cdots + \frac{\$1.15}{(1.134)^{50}} + \cdots + \frac{\$1.15}{(1.134)^{100}} + \cdots \\ &= \$1.01 + \$0.89 + \$0.79 + \cdots + \$0.002 + \cdots + \$0.000004 + \cdots\end{aligned}$$

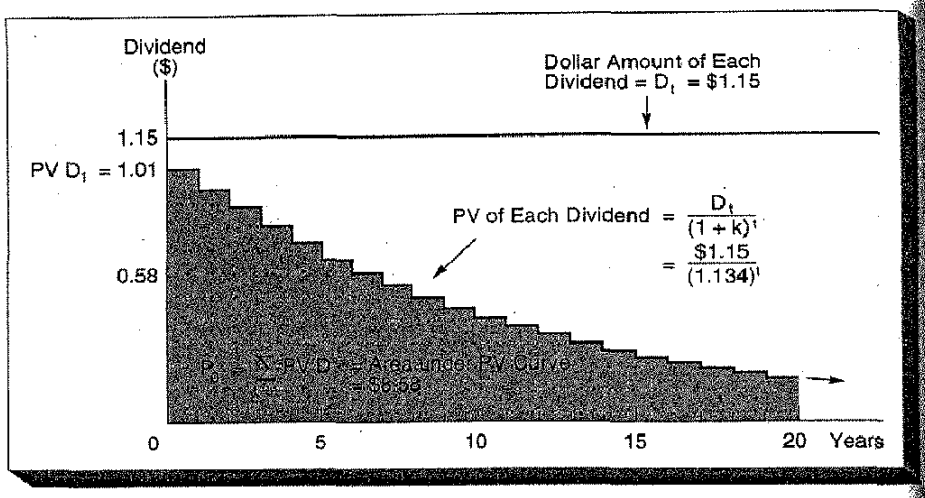
We can also show the zero growth stock in graph form, as in Figure 9-1. The horizontal line shows the constant dividend stream, D_t = \$1.15. The descending step function curve shows the present value of each future dividend. If we extended the analysis on out to infinity and then summed the present values of all the future dividends, the sum would be equal to the value of the stock.

As we saw in Chapter 7, the value of any perpetuity is simply the payment divided by the discount rate, so the value of a zero growth stock reduces to this formula:

$$\hat{P}_0 = \frac{D}{k_s}. \tag{9-2}$$

[6]We should note that investors periodically lose sight of the long-run nature of stocks as investments and forget that in order to sell a stock at a profit, one must find a buyer who will pay the higher price. If you analyzed a stock's value in accordance with Equation 9-1, concluded that the stock's market price exceeded a reasonable value, and then bought the stock anyway, then you would be following the "bigger fool" theory of investment—you think that you may be a fool to buy the stock at its excessive price, but you also think that when you get ready to sell it, you can find someone who is an even bigger fool. The bigger fool theory was widely followed in the summer of 1987, just before the stock market lost more than one-third of its value in the October 1987 crash. Many people think it is back in vogue now, in 1998.

FIGURE 9-1 Present Values of Dividends of a Zero Growth Stock (Perpetuity)



Therefore, the value of our illustrative stock is $8.58:

$$\hat{P}_0 = \frac{\$1.15}{0.134} = \$8.58.$$

If you extended Figure 9-1 on out forever and then added up the present value of each individual dividend, you would end up with the intrinsic value of the stock, $8.58.[7] The actual market price of the stock, P_0, could be greater than, less than, or equal to $8.58 depending on other investors' perceptions of the dividend pattern and riskiness of the stock.

We could transpose the $\hat{P}_0$ and the k_s in Equation 9-2 and solve for k_s to produce Equation 9-3:

$$\hat{k}_s = \frac{D}{P_0}. \qquad (9\text{-}3)$$

We could then look up the price of the stock and the latest dividend, P_0 and D, in the newspaper, and D/P_0 would be the rate of return we could expect to earn if we bought the stock. Since we are dealing with an *expected rate of return,* we put a "hat" on the k value. Thus, if we bought the stock at a price of $8.58 and expected to receive a constant dividend of $1.15, our expected rate of return would be

$$\hat{k}_s = \frac{\$1.15}{\$8.58} = 0.134 = 13.4\%.$$

[7]If you think that having a stock pay dividends forever is unrealistic, then think of it as lasting only for 50 years. Here you would have an annuity of $1.15 per year for 50 years discounted at 13.4 percent. Enter N = 50, I = 13.4, and PMT = 1.15, and then press PV to find the value of the annuity. It is $8.57, which differs by only a penny from that of the perpetuity. Thus, the dividends from Years 51 to infinity contribute almost nothing to the value of the stock.

Normal, or Constant, Growth

Although the zero growth model is applicable to a few companies, the earnings and dividends of most companies are expected to increase over time. Expected growth rates vary from company to company, but dividend growth on average is expected to continue in the foreseeable future at about the same rate as that of the nominal gross domestic product (real GDP plus inflation). On this basis, one might expect the dividend of an average, or "normal," company to grow at a rate of 6 to 8 percent a year. Thus, if a **normal,** or **constant, growth** company's last dividend, which has already been paid, was D_0, its dividend in any future Year t may be forecasted as $D_t = D_0(1 + g)^t$, where g is the constant expected rate of growth. For example, if MicroDrive just paid a dividend of \$1.15 (that is, $D_0 = \$1.15$), and if investors expect an 8 percent growth rate, then the estimated dividend one year hence would be $D_1 = \$1.15(1.08) = \$1.24$; D_2 would be \$1.34; and the estimated dividend five years hence would be

$$D_t = D_0(1 + g)^t = \$1.15(1.08)^5 = \$1.69.$$

Using this method for estimating future dividends, we can determine the current stock value, $\hat{P}_0$, using Equation 9-1 as set forth previously—in other words, we can find the expected future cash flow stream (the dividends), then calculate the present value of each dividend payment, and finally sum these present values to find the value of the stock. Thus, the intrinsic value of the stock is equal to the present value of its expected future dividends.

If g is constant, Equation 9-1 may be rewritten as follows:[8]

$$\begin{aligned}\hat{P}_0 &= \frac{D_0(1+g)^1}{(1+k_s)^1} + \frac{D_0(1+g)^2}{(1+k_s)^2} + \cdots + \frac{D_0(1+g)^\infty}{(1+k_s)^\infty} \\ &= D_0 \sum_{t=1}^{\infty} \frac{(1+g)^t}{(1+k_s)^t} \\ &= \frac{D_0(1+g)}{k_s - g} = \frac{D_1}{k_s - g}. \end{aligned} \tag{9-4}$$

Inserting values into Equation 9-4, we find the value of our illustrative stock to be \$23.00:

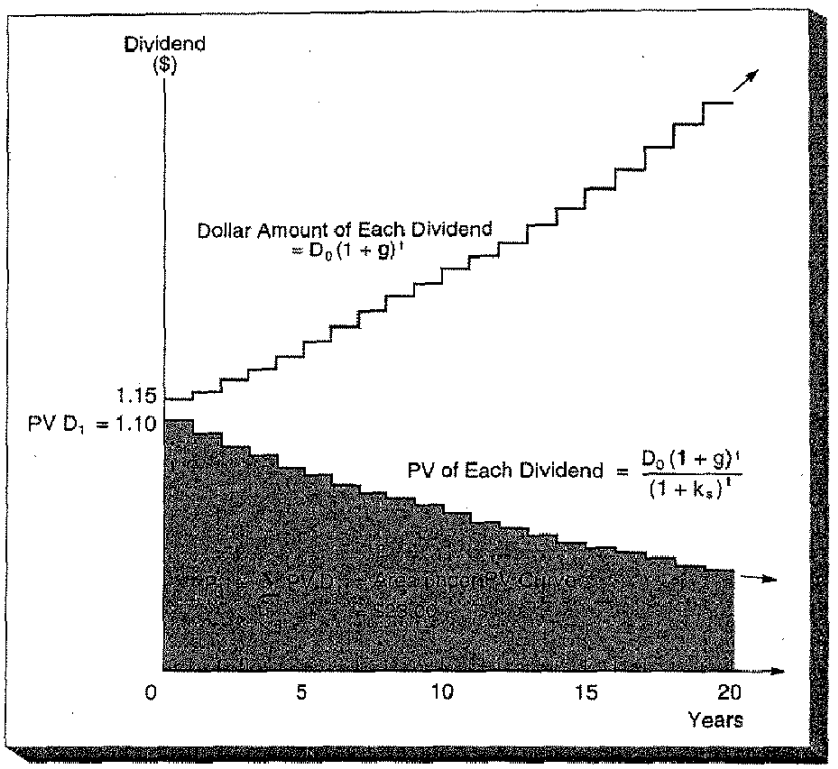
$$\hat{P}_0 = \frac{\$1.15(1.08)}{0.134 - 0.08} = \frac{\$1.242}{0.054} = \$23.00.$$

The **constant growth model** as set forth in the last term of Equation 9-4 is often called the Gordon Model, after Myron J. Gordon, who did much to develop and popularize it.

Note that Equation 9-4 is sufficiently general to encompass the zero growth case described earlier: If growth is zero, this is simply a special case of constant growth, and Equation 9-4 is equal to Equation 9-2. Note also that a necessary condition for the derivation of Equation 9-4 is that k_s be greater than g. Look back at the second form of Equation 9-4. If g is larger than k_s, then $(1 + g)^t/(1 + k_s)^t$ must always be greater than one. In this case, Equation 9-4 is the sum of an infinite number of terms, with each term being a number larger than one. Therefore, if the constant g were greater than k_s, the resulting stock price would be infinite! Since no company is worth an infinite price, it is impossible to have a constant growth rate that is greater than k_s. *So, if you try to*

[8]The last form of Equation 9-4 is derived in Appendix 4A of Eugene F. Brigham and Louis C. Gapenski, *Intermediate Financial Management*, 5th ed. (Fort Worth, Tex.: Dryden Press, 1996). In essence, Equation 9-4 is the sum of a geometric progression, and the final result is the solution value of the progression.

FIGURE 9-2 Present Values of Dividends of a Constant Growth Stock: $D_0 = \$1.15$, $g = 8\%$, $k_s = 13.4\%$



use the constant growth model in a situation where g is greater than k_s, you will violate laws of economics and mathematics, and your results will be both wrong and meaningless.

The concept underlying the valuation process for a constant growth stock is graphed in Figure 9-2. Dividends are growing at the rate $g = 8\%$, but because $k_s > g$, the present value of each future dividend is declining. For example, the dividend in Year 1 is $D_1 = D_0(1 + g)^1 = \$1.15(1.08) = \$1.242$. However, the present value of this dividend, discounted at 13.4 percent, is $PV(D_1) = \$1.242/(1.134)^1 = \$1.095$. The dividend expected in Year 2 grows to $\$1.242(1.08) = \$1.341$, but the present value of this dividend falls to $\$1.04$. Continuing, $D_3 = \$1.449$ and $PV(D_3) = \$0.993$, and so on. Thus, the expected dividends are growing, but the present value of each successive dividend is declining because the dividend growth rate (8%) is less than the rate used for discounting the dividends to the present (13.4%).

If we summed the present values of each future dividend, this summation would be the value of the stock, $\hat{P}_0$. When g is a constant, this summation is equal to $D_1/(k_s - g)$ as shown in Equation 9-4. Therefore, if we extended the lower step function curve in Figure 9-2 on out to infinity and added up the present values of each future dividend, the summation would be identical to the value given by Equation 9-4, \$23.00.

Growth in dividends occurs primarily as a result of growth in *earnings per share (EPS)*. Earnings growth, in turn, results from a number of factors, including (1) inflation

tion, (2) the amount of earnings the company retains and reinvests, and (3) the rate of return the company earns on its equity (ROE). Regarding inflation, if output (in units) is stable, but both sales prices and input costs rise at the inflation rate, then EPS will also grow at the inflation rate. Even without inflation, EPS will also grow as a result of the reinvestment, or plowback, of earnings. If the firm's earnings are not all paid out as dividends (that is, if some fraction of earnings is retained), the dollars of investment behind each share will rise over time, and that should lead to growth in earnings and dividends.

Even though a stock's value is derived from expected dividends, this does not necessarily mean that corporations can increase their stock prices by simply raising the current dividend. Shareholders care about *all* dividends, both current and those expected in the future. Moreover, there is a trade-off between current dividends and future dividends. Companies that pay high current dividends have less money to retain and reinvest in the business, and that lowers the rate of growth in earnings and dividends. So, the issue is this: Do shareholders prefer higher current dividends at the cost of a slower rate of growth in dividends, the reverse, or are stockholders indifferent? As we will see in Chapter 17, there is no simple answer to this question. Shareholders prefer to have the company retain earnings, hence pay less current dividends, if it has highly profitable investment opportunities, but they want the company to pay earnings out if its investment opportunities are poor. Taxes also play a role, as dividends and capital gains are taxed differently, so dividend policy affects investors' taxes. We will consider dividend policy in detail in Chapter 17.

Do Stock Prices Reflect Long-Term or Short-Term Events?

Managers often complain that the stock market is shortsighted and that it cares only about next quarter's performance. Let's use the constant growth model to test this assertion. MicroDrive's most recent dividend was $1.15, and it is expected to grow at a rate of 8 percent per year. Since we know the growth rate, we can forecast the dividends for each of the next five years and then find their present values:

$$
\begin{aligned}
PV &= \frac{D_0(1+g)^1}{(1+k_s)^1} + \frac{D_0(1+g)^2}{(1+k_s)^2} + \frac{D_0(1+g)^3}{(1+k_s)^3} + \frac{D_0(1+g)^4}{(1+k_s)^4} + \frac{D_0(1+g)^5}{(1+k_s)^5} \\
&= \frac{\$1.15(1.08)^1}{(1.134)^1} + \frac{\$1.15(1.08)^2}{(1.134)^2} + \frac{\$1.15(1.08)^3}{(1.134)^3} + \frac{\$1.15(1.08)^4}{(1.134)^4} + \frac{\$1.15(1.08)^5}{(1.134)^5} \\
&= \frac{\$1.242}{(1.134)^1} + \frac{\$1.341}{(1.134)^2} + \frac{\$1.449}{(1.134)^3} + \frac{\$1.565}{(1.134)^4} + \frac{\$1.690}{(1.134)^5} \\
&= 1.095 + 1.043 + 0.993 + 0.946 + 0.901 \\
&\approx \$5.00.
\end{aligned}
$$

Recall that MicroDrive's stock price is $23.00. Therefore, only $5.00, or 22 percent, of the $23.00 stock price is attributable to short-term cash flows. This means that MicroDrive's managers will have a bigger impact on the stock price if they work to increase long-term cash flows rather than focus on short-term flows. This situation holds for most companies. Indeed, a number of professors and consulting firms have used actual company data to show that more than 80 percent of a typical company's stock price is due to cash flows expected more than five years in the future.

This brings up an interesting question. If most of a stock's value is due to long-term cash flows, why do managers and analysts focus so much attention on quarterly earnings? Part of the answer lies in the information conveyed by short-term earnings. For example, if actual quarterly earnings are lower than expected, not because of

fundamental problems but only because a company has increased its R&D expenditures, studies have shown that the stock price probably won't decline and may actually increase. This makes sense, because R&D should increase future cash flows. On the other hand, if quarterly earnings are lower than expected because customers don't like the company's new products, then this new information will have negative implications for future values of g, the long-term growth rate. As we show later in this chapter, even small changes in g can lead to large changes in stock prices. Therefore, while the quarterly earnings itself might not be terribly important, the information they convey about future prospects can be terribly important.

Another reason many managers focus on short-term earnings is that some firms pay managerial bonuses on the basis of current earnings rather than stock prices (which reflect future earnings). For these managers, the concern with quarterly earnings is not due to their effect on stock prices—it's due to current earnings' effect on bonuses.[9]

Expected Rate of Return on a Constant Growth Stock

We can solve Equation 9-4 for k_s, again using the hat to denote that we are dealing with an expected rate of return:[10]

$$\text{Expected rate of return} = \text{Expected dividend yield} + \text{Expected growth rate, or capital gains yield}$$

$$\hat{k}_s = \frac{D_1}{P_0} + g. \tag{9-5}$$

Thus, if you buy a stock for a price $P_0 = \$23$, and if you expect the stock to pay a dividend $D_1 = \$1.242$ one year from now and to grow at a constant rate $g = 8\%$ in the future, then your expected rate of return will be 13.4 percent:

$$\hat{k}_s = \frac{\$1.242}{\$23} + 8\% = 5.4\% + 8\% = 13.4\%.$$

In this form, we see that $\hat{k}_s$ is the *expected total return* and that it consists of an *expected dividend yield,* $D_1/P_0 = 5.4\%$, plus an *expected growth rate or capital gains yield,* $g = 8\%$.

Suppose this analysis had been conducted on January 1, 1999, so $P_0 = \$23$ is the January 1, 1999, stock price, and $D_1 = \$1.242$ is the dividend expected at the end of 1999. What is the expected stock price at the end of 2000? We would again apply Equation 9-4, but this time we would use the year-end dividend, $D_2 = D_1(1 + g) = \$1.242(1.08) = \$1.3414$:

$$\hat{P}_{12/31/99} = \frac{D_{2000}}{k_s - g} = \frac{\$1.3414}{0.134 - 0.08} = \$24.84.$$

Now, notice that \$24.84 is 8 percent greater than P_0, the \$23 price on January 1, 1999:

$$\$23(1.08) = \$24.84.$$

[9]Many apparent puzzles in finance can be explained either by managerial compensation systems or by peculiar features of the Tax Code. So, if you can't explain a firm's behavior in terms of economic logic, look at bonuses or taxes as possible explanations.

[10]The k_s value in Equation 9-4 is a *required* rate of return, but when we transform to obtain Equation 9-5 we are finding an *expected* rate of return. The transformation requires that $k_s = \hat{k}_s$, which holds if the stock market is in equilibrium, a condition that will be discussed later in the chapter.

Thus, we would expect to make a capital gain of \$24.84 − \$23.00 = \$1.84 during 1999, which would provide a capital gains yield of 8 percent:

$$\text{Capital gains yield}_{1999} = \frac{\text{Capital gain}}{\text{Beginning price}} = \frac{\$1.84}{\$23.00} = 0.08 = 8\%.$$

We could extend the analysis on out, and in each future year the expected capital gains yield would always equal g, the expected dividend growth rate.

Continuing, the dividend yield in 2000 could be estimated as follows:

$$\text{Dividend yield}_{2000} = \frac{D_{2000}}{\hat{P}_{12/31/99}} = \frac{\$1.3414}{\$24.84} = 0.054 = 5.4\%.$$

The dividend yield for 2001 could also be calculated, and again it would be 5.4 percent. Thus, *for a constant growth stock,* the following conditions must hold:

1. The dividend is expected to grow forever at a constant rate, g.
2. The stock price is expected to grow at that same rate.
3. The expected dividend yield is a constant.
4. The expected capital gains yield is also a constant, and it is equal to g.
5. The expected total return, $\hat{k}_s$, is equal to the expected dividend yield plus the expected growth rate: $\hat{k}_s$ = dividend yield + g.

The term *expected* should be clarified—it means expected in a probabilistic sense, as the statistically expected outcome. Thus, if we say the growth rate is expected to remain constant at 8 percent, we mean that the best prediction for the growth rate in any future year is 8 percent, not that we literally expect the growth rate to be exactly 8 percent in each future year. In this sense, the constant growth assumption is reasonable for many large, mature companies.

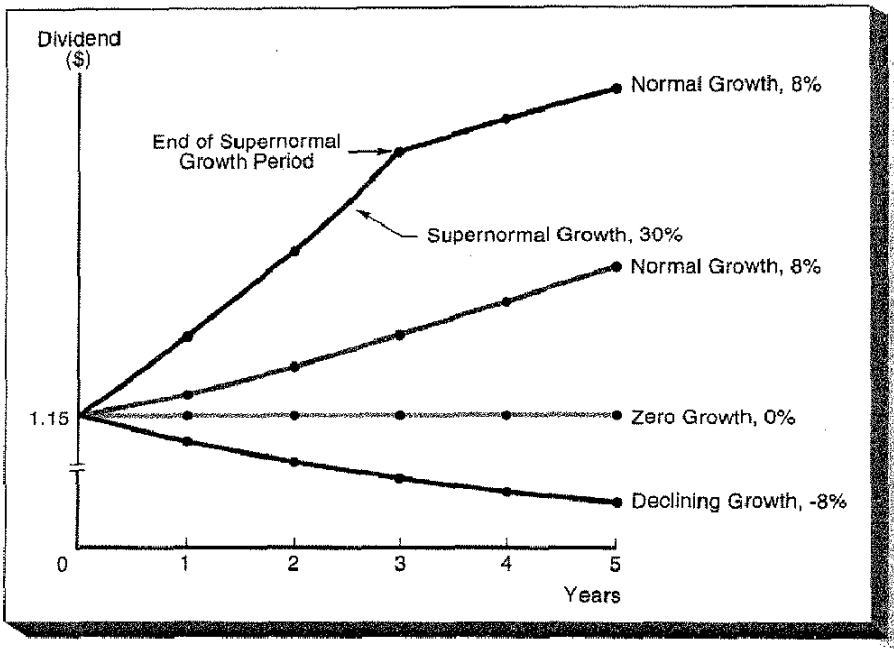
Supernormal, or Nonconstant, Growth

Firms typically go through *life cycles.* During the early part of their lives, their growth is much faster than that of the economy as a whole; then they match the economy's growth; and finally their growth is slower than that of the economy.[11] Automobile manufacturers in the 1920s and computer software firms such as Microsoft in the 1990s are examples of firms in the early part of the cycle; these firms are called **supernormal,** or **nonconstant, growth** firms. Figure 9-3 illustrates nonconstant growth and also compares it with normal growth, zero growth, and negative growth.[12]

[11]The concept of life cycles could be broadened to *product cycle,* which would include both small startup companies and large companies like Procter & Gamble, which periodically introduce new products that give sales and earnings a boost. We should also mention *business cycles,* which alternately depress and boost sales and profits. The growth rate just after a major new product has been introduced, or just after a firm emerges from the depths of a recession, is likely to be much higher than the "expected long-run average growth rate," which is the proper number for a DCF analysis.

[12]A negative growth rate indicates a declining company. A mining company whose profits are falling because of a declining ore body is an example. Someone buying such a company would expect its earnings, and consequently its dividends and stock price, to decline each year, and this would lead to capital losses rather than capital gains. Obviously, a declining company's stock price will be relatively low, and its dividend yield must be high enough to offset the expected capital loss and still produce a competitive total return. Students sometimes argue that they would not be willing to buy a stock whose price was expected to decline. However, if the annual dividends are large enough to *more than offset* the falling stock price, the stock could still provide a good return.

FIGURE 9-3 Illustrative Dividend Growth Rates



In the figure, the dividends of the supernormal growth firm are expected to grow at a 30 percent rate for three years, after which the growth rate is expected to fall to 8 percent, the assumed average for the economy. The value of this firm, like any other, is the present value of its expected future dividends as determined by Equation 9-1. In the case in which D_t is growing at a constant rate, we simplified Equation 9-1 to $\hat{P}_0 = D_1/(k_s - g)$. In the supernormal case, however, the expected growth rate is not a constant—it declines at the end of the period of supernormal growth.

To find the value of such a stock, or of any nonconstant growth stock when the growth rate will eventually stabilize, we proceed in three steps:

1. Find the PV of the dividends during the period of nonconstant growth.
2. Find the price of the stock at the end of the nonconstant growth period, at which point it has become a constant growth stock, and discount this price back to the present.
3. Add these two components to find the intrinsic value of the stock, $\hat{P}_0$.

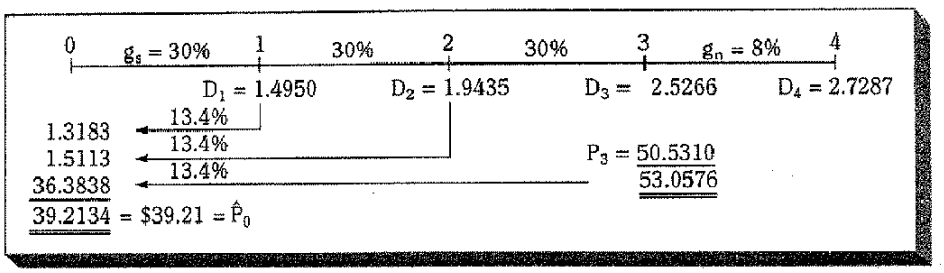
Figure 9-4 can be used to illustrate the process for valuing nonconstant growth stocks, assuming the following five facts exist:

k_s = stockholders' required rate of return = 13.4%. This rate is used to discount the cash flows.

N = years of supernormal growth = 3.

g_s = rate of growth in both earnings and dividends during the supernormal growth period = 30%. (Note: The growth rate during the supernormal growth period

FIGURE 9-4 Process for Finding the Value of a Supernormal Growth Stock



NOTES TO FIGURE 9-4:

Step 1. Calculate the dividends expected at the end of each year during the supernormal growth period. Calculate the first dividend, $D_1 = D_0(1 + g_s) = \$1.15(1.30) = \$1.4950$. Here g_s is the growth rate during the three-year supernormal growth period, 30 percent. Show the \$1.4950 on the time line as the cash flow at Time 1. Then, calculate $D_2 = D_1(1 + g_s) = \$1.4950(1.30) = \$1.9435$, and then $D_3 = D_2(1 + g_s) = \$1.9435(1.30) = \$2.5266$. Show these values on the time line as the cash flows at Time 2 and Time 3. Note that D_0 is used only to calculate D_1.

Step 2. The price of the stock is the PV of dividends from Time 1 to infinity, so in theory we could project each future dividend, with the normal growth rate, g_n = 8%, used to calculate D_4 and subsequent dividends. However, we know that after D_3 has been paid, which is at Time 3, the stock becomes a constant growth stock. Therefore, we can use the constant growth formula to find $\hat{P}_3$, which is the PV of the dividends from Time 4 to infinity as evaluated at Time 3.

First, we determine $D_4 = \$2.5266(1.08) = \$2.7287$ for use in the formula, and then we calculate $\hat{P}_3$ as follows:

$$\hat{P}_3 = \frac{D_4}{k_s - g_n} = \frac{\$2.7287}{0.134 - 0.08} = \$50.5310.$$

We show this \$50.5310 on the time line as a second cash flow at Time 3. The \$50.5310 is a Time 3 cash flow in the sense that the owner of the stock could sell it for \$50.5310 at Time 3 and also in the sense that \$50.5310 is the present value of the dividend cash flows from Time 4 to infinity. Note that the *total cash flow* at Time 3 consists of the sum of $D_3 + \hat{P}_3 = \$2.5266 + \$50.5310 = \$53.0576$.

Step 3. Now that the cash flows have been placed on the time line, we can discount each cash flow at the required rate of return, k_s = 13.4%. We could discount each flow by dividing by $(1.134)^t$, where t = 1 for Time 1, t = 2 for Time 2, and t = 3 for Time 3. This produces the PVs shown to the left below the time line, and the sum of the PVs is the value of the supernormal growth stock, \$39.21.

With a financial calculator, you can find the PV of the cash flows as shown on the time line with the cash flow (CFLO) register of your calculator. Enter 0 for CF_0 because you get no cash flow at Time 0, CF_1 = 1.495, CF_2 = 1.9435, and CF_3 = 2.5266 + 50.531 = 53.0576. Then enter I = 13.4, and press the NPV key to find the value of the stock, \$39.21.

could vary from year to year. Also, there could be several different supernormal growth periods, e.g., 30% for three years, then 20% for three years, and then a constant 8%.) This rate is shown directly on the time line.

g_n = rate of normal, constant growth after the supernormal period = 8%. This rate is also shown on the time line, between Periods 3 and 4.

D_0 = last dividend the company paid = \$1.15.

The valuation process as diagrammed in Figure 9-4 is explained in the steps set forth below the time line. The value of the supernormal growth stock is calculated to be \$39.21.

SELF-TEST QUESTIONS

Explain the following statement: "Whereas a bond contains a promise to pay interest, common stock typically provides an expectation of but no promise of dividends plus capital gains."

Are stock values based more on current earnings or long-term earnings forecasts?

What are the two parts of a stock's expected total return?

Write out and explain the valuation model for a zero growth stock.

Write out and explain the valuation model for a constant growth stock.

How does one calculate the capital gains yield and the dividend yield of a stock?

Explain how one would find the value of a supernormal growth stock.

VALUING THE ENTIRE CORPORATION

In the previous section, we presented several equations for valuing a firm's common stock. These equations had one common element: They all assumed that the firm is currently paying a dividend. But consider the situation of a startup company formed to develop and market a new product. Such a company generally expects to have low sales during its first few years as it develops and begins to market its product. Then, if the product catches on, sales will grow rapidly for several years. For example, Compaq Computer had just three employees when it was founded in 1982. Its first year was devoted to product development, so 1982 sales were zero. In early 1983, however, Compaq introduced its personal computer, and its 1983 sales hit $111 million, a record first-year volume for any new firm. Two years later, Compaq was included in *Fortune's* 500 largest U.S. industrial firms. Obviously, Compaq was more successful than most new businesses, but growth rates of 100, 500, or even 1,000 percent are not uncommon during firms' early years.

Growing sales require additional assets—Compaq could not have grown as it did without increasing its assets. Moreover, asset growth must be financed by increasing some liability and/or equity account. Small firms can often obtain some bank credit, but they must maintain a reasonable balance between debt and equity. Thus, additional bank borrowings require increases in equity, but small firms have limited access to the stock market. Moreover, even if they can sell stock, their owners are often reluctant to do so for fear of losing voting control. Therefore, the best source of equity for most small businesses is from retaining earnings, so most small firms pay no dividends during their rapid growth years. Eventually, most successful firms do pay dividends, with dividends growing rapidly at first but then slowing down as the firm approaches maturity.

Although most larger firms do pay a dividend, some firms, even highly profitable ones such as Microsoft, have never paid a dividend. How can the value of such a company be determined? Similarly, suppose you start a business, and someone offers to buy it from you. How could you determine its value, or that of any privately held business? Or suppose you work for a company with a number of divisions. How could you determine the value of one particular division which the company wants to sell? In none of these cases could you use the dividend growth model. However, you could use the total **company,** or **corporate, valuation model.**

The Corporate Value Model

Tables 9-2 and 9-3 contain the actual 1998 and projected 1999 to 2002 financial statements for MagnaVision Inc., which produces optical systems for use in medical photography. (See Chapter 14 for more details on how to project financial statements.)

TABLE 9-2 MagnaVision Inc.: Income Statements (Millions of Dollars Except for Per Share Data)

	Actual	Projected			
	1998	1999	2000[b]	2001	2002
Net sales	$700.0	$850.0	$1,000.0	$1,100.0	$1,155.0
Costs (except depreciation)	$599.0	$734.0	$ 911.0	$ 935.0	$ 982.0
Depreciation	28.0	31.0	34.0	36.0	38.0
Total operating costs	$627.0	$765.0	$ 945.0	$ 971.0	$1,020.0
Earnings before interest and taxes (EBIT)	$ 73.0	$ 85.0	$ 55.0	$ 129.0	$ 135.0
Less: Net interest[a]	13.0	15.0	16.0	17.0	19.0
Earnings before taxes	$ 60.0	$ 70.0	$ 39.0	$ 112.0	$ 116.0
Taxes (40%)	24.0	28.0	15.6	44.8	46.4
Net income before preferred dividends	$ 36.0	$ 42.0	$ 23.4	$ 67.2	$ 69.6
Preferred dividends	6.0	7.0	7.4	8.0	8.3
Net income available for common dividends	$ 30.0	$ 35.0	$ 16.0	$ 59.2	$ 61.3
Common dividends	—	—	—	$ 44.2	$ 45.3
Addition to retained earnings	$ 30.0	$ 35.0	$ 16.0	$ 15.0	$ 16.0
Number of shares	100	100	100	100	100
Dividends per share	—	—	—	$0.442	$0.453

[a]"Net interest" is interest paid on debt less interest earned on marketable securities. Both items could be shown separately on the income statements, but for this example we combine them and show net interest.

[b]Net income is projected to decline in 2000. This is due to a projected cost for a one-time marketing program in that year.

Growth has been rapid in the past, but the market is becoming saturated, so the sales growth rate is expected to decline from 21 percent in 1999 to a sustainable rate of 5 percent in 2002 and beyond. Profit margins are expected to improve as the production process becomes more efficient and because MagnaVision will no longer be incurring marketing costs associated with the introduction of a major product. All items on the financial statements are projected to grow at a 5 percent rate after the year 2002. Notice that the company does not pay a dividend, but it is expected to start paying out about 75 percent of its earnings beginning in 2001. (Chapter 17 explains in more detail how companies decide how much to pay out in dividends.)

In Chapters 1 and 2 we explained that a firm's value is determined by its ability to generate cash flow, both now and in the future. Therefore, MagnaVision's value can be calculated as the present value of its expected future free cash flows (FCF) from operations, discounted at its cost of capital, k_c, plus the value of its nonoperating assets. Here is the equation for the value of operations, or the firm's value as a going concern:

$$\begin{aligned}\text{Value of operations} = V_{op} &= \text{PV of expected future free cash flow}\\ &= \frac{FCF_1}{(1+k_c)^1} + \frac{FCF_2}{(1+k_c)^2} + \cdots + \frac{FCF_\infty}{(1+k_c)^\infty} \qquad \textbf{(9-6)}\\ &= \sum_{t=1}^{\infty} \frac{FCF_t}{(1+k_c)^t}.\end{aligned}$$

TABLE 9-3 MagnaVision Inc.: Balance Sheets (Millions of Dollars)

	Actual	Projected			
	1998	1999	2000	2001	2002
Assets					
Cash	$ 17.0	$ 20.0	$ 22.0	$ 23.0	$ 24.1
Marketable securities[a]	63.0	70.0	80.0	84.0	88.2
Accounts receivable	85.0	100.0	110.0	116.0	121.8
Inventories	170.0	200.0	220.0	231.0	242.6
Total current assets	$ 335.0	$ 390.0	$ 432.0	$ 454.0	$ 476.7
Net plant and equipment	279.0	310.0	341.0	358.0	376.0
Total assets	$ 614.0	$ 700.0	$ 773.0	$ 812.0	$ 852.6
Liabilities and Equity					
Accounts payable	$ 17.0	$ 20.0	$ 22.0	$ 23.0	$ 24.1
Notes payable	123.0	140.0	160.0	168.0	176.4
Accruals	43.0	50.0	55.0	58.0	61.0
Total current liabilities	$ 183.0	$ 210.0	$ 237.0	$ 249.0	$ 261.5
Long-term bonds	$ 124.0	$ 140.0	$ 160.0	$ 168.0	$ 176.4
Preferred stock	62.0	70.0	80.0	84.0	88.2
Common stock[b]	200.0	200.0	200.0	200.0	200.0
Retained earnings	45.0	80.0	96.0	111.0	127.5
Common equity	$ 245.0	$ 280.0	$ 296.0	$ 311.0	$ 327.5
Total liabilities and equity	$ 614.0	$ 700.0	$ 773.0	$ 812.0	$ 852.6

NOTES:

[a]All assets except marketable securities are operating assets required to support sales. The marketable securities are financial assets not required in operations.

[b]Par plus paid-in capital.

In Chapter 10 we explain how to estimate the cost of capital, but for now we assume that MagnaVision's cost of capital is 10.84 percent. To find its going concern value, we use an approach similar to the nonconstant dividend growth model, proceeding as follows:

1. Assume that the firm will experience nonconstant growth for N years, after which it will grow at some constant rate.
2. Calculate the expected free cash flow for each of the N nonconstant growth years and find the PV of these cash flows.
3. Recognize that after Year N growth will be constant, so we can use the constant growth formula to find the firm's value at Year N. This "terminal value" is the sum of the PVs for N + 1 and all subsequent years, discounted back to Year N. Then, the Year N value must be discounted back to the present to find its PV at Year 0.
4. Now sum all the the PVs, those of the annual free cash flows during the nonconstant period plus the PV of the terminal value, to find the firm's value of operations. The going concern value, when added to the value of the nonoperating assets, is the total value of the firm.

TABLE 9-4 Calculating MagnaVision's Expected Free Cash Flow (Millions of Dollars)

	Actual 1998	Projected 1999	2000	2001	2002
Calculation of Free Cash Flow					
1. Required net operating working capital	$212.00	$250.00	$275.00	$289.00	$303.00
2. Required net plant and equipment	279.00	310.00	341.00	358.00	376.00
3. Required net operating assets	$491.00	$560.00	$616.00	$647.00	$679.00
4. Required net new investment in operating assets = change in net operating assets from previous year		69.00	56.00	31.00	32.00
5. NOPAT [Net operating profit after taxes = EBIT × (1 − Tax rate)]		$ 51.00	$ 33.00	$ 77.40	$ 81.00
6. Less: Required investment in operating assets		69.00	56.00	31.00	32.00
7. Free cash flow		($ 18.00)	($ 23.00)	$ 46.40	$ 49.00

Note: NOPAT declines in 2000 because of a marketing expenditure projected for that year. See Note b in Table 9-2.

Table 9-4 calculates free cash flow for each year, using procedures discussed in Chapter 2. Line 1, with data for 1998 from the balance sheets in Table 9-3, shows the required net operating working capital, or operating current assets minus operating current liabilities, for 1998:

$$\text{Required net operating working capital} = \begin{pmatrix}\text{Cash} + \\ \text{Accounts receivable} \\ + \text{Inventories}\end{pmatrix} - \begin{pmatrix}\text{Accounts} \\ \text{payable} + \\ \text{Accruals}\end{pmatrix}$$

$$= (\$17.00 + \$85.00 + \$170.00) - (\$17.00 + \$43.00)$$

$$= \$212.00.$$

Line 2 shows required net plant and equipment, and Line 3, which is the sum of Lines 1 and 2, shows the required net operating assets, sometimes called net operating capital. For 1998, net operating capital is $212 + $279 = $491 million.

Line 4 shows the required net annual addition to operating assets, found as the change in net operating assets from the previous year. For 1999, the required net investment in operating assets is $560 − $491 = $69 million.

Line 5 shows NOPAT, or net operating profit after taxes. Note that EBIT is operating earnings *before* taxes, while NOPAT is operating earnings *after* taxes. Therefore, NOPAT = EBIT(1 − T). With 1999 EBIT of $85 as shown in Table 9-2 and a tax rate of 40 percent, NOPAT as projected for 1999 is $51 million:

$$\text{NOPAT} = \text{EBIT}(1 - T) = \$85(1.0 - 0.4) = \$51 \text{ million.}$$

Although MagnaVision's operating assets are projected to produce $51 million of after-tax profits in 1999, the company must invest $69 million in new assets in 1999. Therefore, the free cash flow for 1999, shown on Line 7, is a negative $18 million:

$$\text{Free cash flow (FCF)} = \$51 - \$69 = -\$18.00 \text{ million.}$$

This negative free cash flow in the early years is typical for young, high-growth companies. Even though net operating profit after taxes (NOPAT) is positive in all years, free cash flow is negative because of the need to invest in operating assets. The negative free

cash flow means the company will have to obtain new funds from investors, and the balance sheets in Table 9-3 show that notes payable, long-term bonds, and preferred stock all increase from 1998 to 1999. Stockholders will also help fund MagnaVision's growth—they will receive no dividends until 2001, so all of the net income from 1999 to 2001 will be reinvested. However, as growth slows, free cash flow will become positive, and MagnaVision plans to use some of its FCF to pay dividends beginning in 2001.[13]

A variant of the constant growth dividend model (Equation 9-4) is shown below as Equation 9-7. This equation can be used to find the value of MagnaVision's operations once its free cash flows stabilize and begin to grow at a constant rate:

$$V_{op(\text{at time } N)} = \sum_{t=N+1}^{\infty} \frac{FCF_t}{(1+k_c)^{t-N}} = \frac{FCF_N(1+g)}{k_c - g} = \frac{FCF_{N+1}}{k_c - g}. \tag{9-7}$$

Based on a 10.84 percent cost of capital, a $49 million free cash flow in 2002, and a 5 percent growth rate, the value of MagnaVision's operations as of December 31, 2002, is forecasted to be $880.99 million:

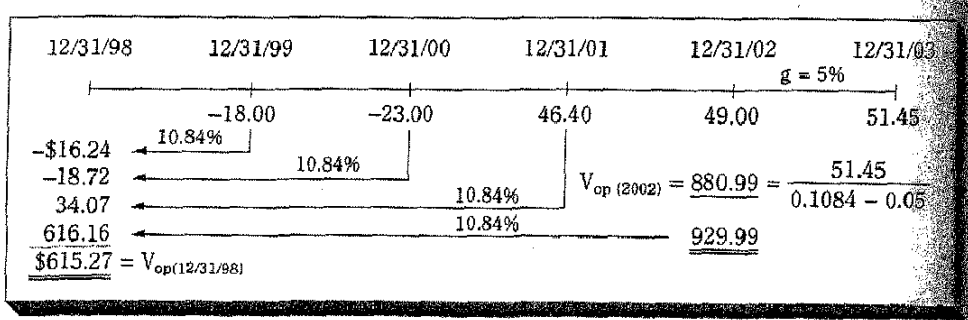
$$V_{op(12/31/02)} = \frac{FCF_{12/31/02}(1+g)}{k_c - g} = \frac{FCF_{12/31/03}}{k_c - g} = \frac{\$49(1+0.05)}{0.1084 - 0.05} = \frac{\$51.45}{0.1084 - 0.05} = \$880.99. \tag{9-7a}$$

This $880.99 million figure is called the company's **terminal,** or **horizon, value,** because it is the value at the end of the forecast period. Moreover, this is the amount that MagnaVision could expect to receive if it sold its operating assets on December 31, 2002.

Figure 9-5 shows the free cash flow for each year during the nonconstant growth period, along with the value of operations in 2002, at the end of the nonconstant growth period. To find the value of operations as of "today," December 31, 1998, we find the PV of each annual cash flow in Figure 9-5, discounting at the 10.84 percent cost of

[13]MagnaVision plans to increase its debt and preferred stock each year so as to maintain a constant capital structure. We discuss capital structure in detail in Chapters 14, 15, and 16.

FIGURE 9-5 Process for Finding the Value of Operations for a Nonconstant Growth Company



capital. The sum of the PVs is approximately $615 million, and it represents an estimate of the price MagnaVision could expect to receive if it sold its operating assets today, December 31, 1998.

The total value of any company is the value of its operations plus the value of its nonoperating assets. As the December 31, 1998, balance sheet in Table 9-3 shows, MagnaVision had $63 million of marketable securities on that date. Unlike operating assets, we do not have to calculate a present value for marketable securities because short-term financial assets as reported on the balance sheet are at, or close to, their market value. Therefore, MagnaVision's total value on December 31, 1998, is $615.27 + $63.00 = $678.27 million.

If the company's total value on December 31, 1998, is $678.27 million, what is the value of its common equity? First, note that notes payable and long-term debt total $123 + $124 = $247 million, and these securities have the first claim on assets and income. (Accounts payable and accruals were netted out earlier.) Next, the preferred stock has a claim of $62 million, and it also ranks above the common. Therefore, the value left for common stockholders is $678.27 − $247 − $62 = $369.27 million.

Figure 9-6 is a bar chart which analyzes the sources of MagnaVision's value. The left bar shows the company's total value as the sum of its nonoperating assets plus its going concern value. Next, what is the claim of each class of investors on that total value? This breakdown is shown in the middle bar. Debtholders have the highest priority claim, and MagnaVision owes $123 million on notes payable and $124 million on long-term bonds, for a total of $247 million. The preferred stockholders have the next claim, $62 million. The remainder of the value belongs to the common equity, and it

FIGURE 9-6 MagnaVision's Value as of December 31, 1998

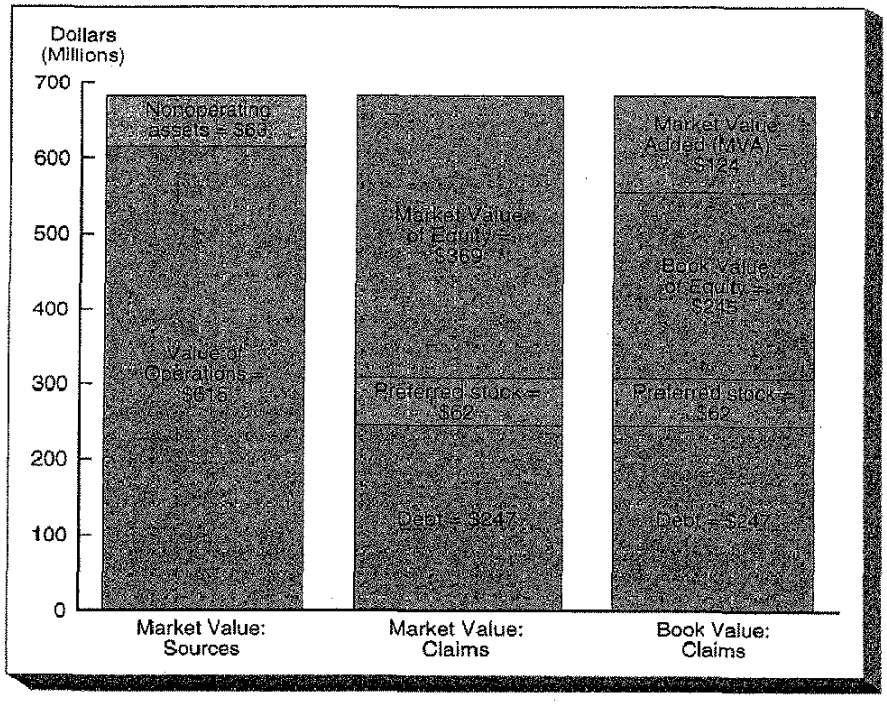


TABLE 9-5 Finding the Value of MagnaVision's Stock (Millions of Dollars Except for Per Share Data)

1. Value of operations (net of payables and accruals)	$615.27
2. Plus value of nonoperating assets	63.00
3. Total market value of the firm	$678.27
4. Less: Value of debt	247.00
Value of preferred stock	62.00
5. Value of common equity	$369.27
6. Divide by number of shares	100
7. Value per share	$3.69

amounts to $678.27 − $247.00 − $62.00 = $369.27 million.[14] Finally, the bar on the right side of Figure 9-6 divides the market value of the equity into the book value, which represents the actual investment stockholders have made, and the additional market value added (MVA) by management.

Table 9-5 summarizes the calculations used to find MagnaVision's stock value. There are 100 million shares outstanding, and their total value is $369.27 million. Therefore, the value of a single share is $3.69 ($369.27/100 = $3.69).

The Dividend Growth Model Applied to MagnaVision

MagnaVision has not yet begun to pay dividends. However, as shown in Table 9-2, a cash dividend of $0.442 per share is forecasted for 2001, the dividend is expected to grow by about 2.5 percent in 2002, and then dividends are expected to grow at a constant 5 percent rate thereafter. In Chapter 10 we explain how to estimate each component cost of capital; for now we simply assume that the firm's cost of equity is 14 percent. In this situation, we can apply the nonconstant dividend growth model as developed earlier in the chapter. Figure 9-7 shows that the value of MagnaVision's stock, based on the model, is $3.70 per share, which is identical to the value found using the total corporate model except for rounding errors.

Comparing the Total Company and Dividend Growth Models

Since the total company and dividend growth models give the same answer, does it matter which model you choose? In general, it does. For example, if you were a financial analyst estimating the values of mature companies whose dividends are expected to grow steadily in the future, it would probably be more efficient to use the dividend growth model. Here you would only need to estimate the growth rate in dividends, not the entire set of pro forma financial statements. Chapter 10 explains some techniques for estimating growth rates.

However, if a company is paying a dividend but is still in the high-growth stage of its life cycle, you would need to project the future financial statements before you could make a reasonable estimate of future dividends. Then, since you would have already estimated future financial statements, it would be a toss-up as to whether the total company model or the dividend growth model would be easier to apply. Intel, which

[14]Rather than subtracting the book values of debt and preferred stock, it would be better to subtract their market values. In most cases, including this one, the book values of fixed income securities are close to their market values, and when this is true, one can simply work with book values.